MODEL OF GAS FLOW INSIDE A PLASMA THRUSTER

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Summary Stationary Plasma Thrusters (SPT) are high-efficiency electric rocket engines that are ideally suited to spacecraft orbit transfer and positioning missions. By harnessing the power resources available on-board to accelerate a plasma, these engines allow substantial savings on the propellant mass in comparison with chemical rocket engines.

Further improvements of their performance remain hindered, however, by the poor understanding of the underlying physics. With this incentive, a large number of numerical models of the discharge are being developed [1, 2, 3, 4]. In such models, the dynamic of neutral particles (non-ionized propellant) is a key element in the overall description of the discharge. It has been shown in particular that the flow of neutral gas is directly involved in the so called "breathing mode", a low frequency longitudinal mode that strongly affects the operation of the engine.

Because of the complexity of SPT plasmas, many different modeling approaches have been adopted to investigate different time scales and phenomena. In many models of discharge, a fully kinetic description of the gas flow is either not acceptable due to computational constraints or not necessary in view of the low accuracy inherent to the plasma description. Because the Knudsen number of the flow of neutral gas is much larger than one, it is commonly assumed that the mean axial velocity of neutrals is constant through the channel [3, 2]. This assumption is challenged, however, by ionization which acts as a selection process by removing slow neutrals, thus allowing only fast neutrals to reach the exit. Additionally, the diffuse reflections of neutrals on the walls gives rise to a reverse flow and makes the mean axial velocity smaller upstream than downstream.

The goal of this study is to propose a simple, yet more realistic alternative to the classical monokinetic model, using one-dimensional macroscopic equations that account for the phenomena mentioned above.

MAIN ASSUMPTIONS

The annular channel is bounded by two coaxial cylinders of radii R_{in} , R_{out} and of length L. The gas is fed through a circular slit of radius $(R_{in} + R_{out})/2$ located at the closed extremity of the channel (at axial position $z = 0$).

Because the Knudsen number is typically of the order of 100, the flow is supposed collisionless. The gas flow accounts for both ground level and excited neutrals; once a particle is ionized, it is removed from the gas flow. We assume that the distribution function at the gas feed is half-Maxwellian (i.e. consists only of particles with a positive axial velocity) with a temperature T_c equal to that of the channel.

Typical density and mean velocity maps of neutrals in such a channel are represented on Fig. 1.

DERIVATION OF ONE-DIMENSIONAL MACROSCOPIC EQUATIONS

Preliminary result for a stationary flow with smooth walls and ionization

If particles are specularly reflected at the walls and if the ionization frequency ν_i is a function of z only, the distribution of neutrals as a function of their axial velocity integrated over the cross-section evolves along the z axis as

$$f_z(z,\, v_z) = \frac{\dot{M}}{\kappa T_c A} \exp\left(-\frac{m v_z^2}{2\kappa T_c} - \frac{1}{v_z}\int_0^z \nu_i(z)\, dz\right) \qquad \text{with } v_z > 0\,. \tag{1}$$

where $\dot{M}$ is the mass flow rate, m the mass of a neutral, A the cross-section of the channel and T_c the temperature of the channel. Defining now the axial velocity dispersion as $D_z(z) = \frac{1}{n}\int (v_z - V_z)^2\, f_z(z,\, v_z)\, dv_z$ where n is the density and V_z the mean axial velocity of the flow, it can be shown after some manipulations that for any $z > 0$, D_z is bounded by

$$\frac{1}{3}\frac{\kappa T_c}{m} < D_z < \left(1 - \frac{2}{\pi}\right)\frac{\kappa T_c}{m} < 0.364\,\frac{\kappa T_c}{m}\,. \tag{2}$$

Model for rough walls and ionization

Particles which collide with the walls are now assumed to be thermalized. The *primary flow* directly injected by the gas feed and the *secondary flow* of particles thermalized by collisions with the walls are thus distinguished.

The amount of particles transfered at the walls from the primary to the secondary flow can be estimated using view factors [5]. Assumed that in the primary flow and at a given position z the ratio between the axial flux and the flux to the walls is independent of ionization and only given by geometrical factors, this ratio is then $\zeta(z) = z/\left[(R_{out} - R_{in})/4 + z^2\right]$. Additionally, the neutral-wall collision frequency ν_{nw} for the secondary flow is straightforwardly obtained by assuming that it is in thermal equilibrium with the channel walls, i.e. $\nu_{nw} = \sqrt{2\kappa T_c/\pi m}/(R_{out} - R_{in})$.

The macroscopic equations for the primary flow are then derived by integration of Boltzmann equations using the approximation $D_z \approx \kappa T_c/3m$ suggested by Eq. (2). For the secondary flow, a reasonable assumption to close the equation system consists in stating that the thermalization with the walls is perfect ($D_z = \kappa T_c/m$).

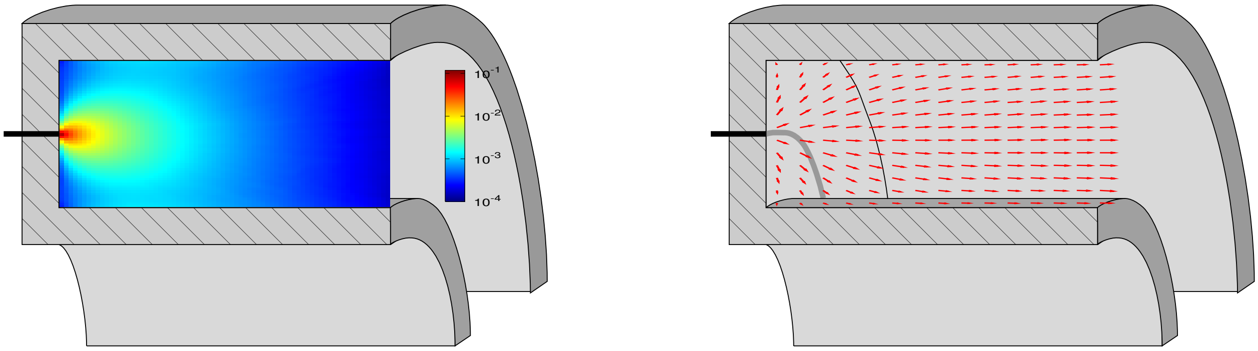

Figure 1. Stationary density (left) and mean velocity (right) maps of neutrals inside the channel for a uniform ionization frequency (results obtained with a fully kinetic Monte-Carlo simulation). $T_c = 800$K and $\nu_i = 30$kHz.

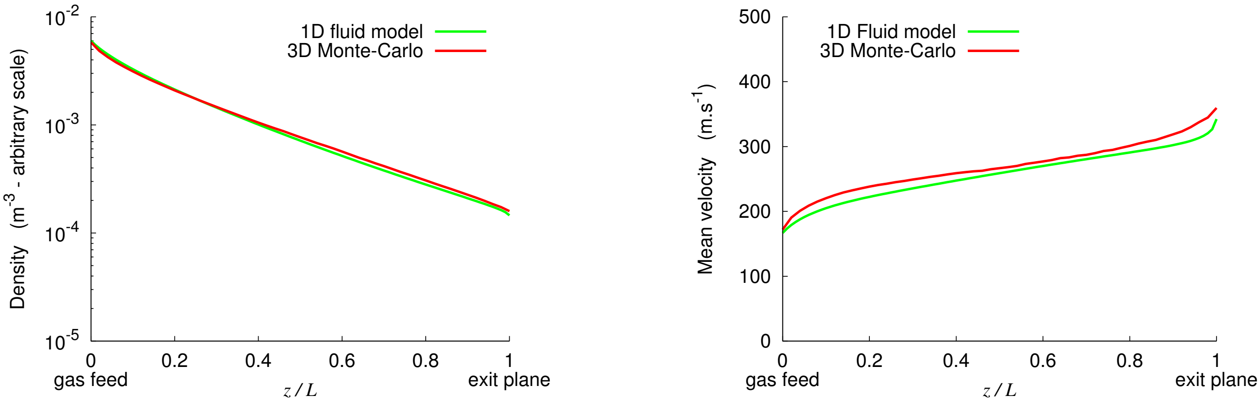

Figure 2. Section-averaged stationary profiles of density (left) and mean velocity (right) in the channel obtained with a 3D fully kinetic simulation and with the macroscopic model, for a uniform ionization frequency. $T_c = 800$K and $\nu_i = 30$kHz.

With these assumptions, the final equations read:

$$\frac{\partial n_1}{\partial t} + \frac{\partial (n_1 V_{1z})}{\partial z} = -\nu_i \, n_1 - \zeta(z) \, n_1 V_{1z} \, , \tag{3}$$

$$\frac{\partial (n_1 V_{1z})}{\partial t} + \frac{\partial (n_1 V_{1z}^2 + n_1 \frac{\kappa T_c}{3m})}{\partial z} = -\nu_i \, n_1 V_{1z} - \zeta(z) \, n_1 V_{1z}^2 \, . \tag{4}$$

$$\frac{\partial n_2}{\partial t} + \frac{\partial (n_2 V_{2z})}{\partial z} = -\nu_i \, n_2 + \zeta(z) \, n_1 V_{1z} \, , \tag{5}$$

$$\frac{\partial (n_2 V_{2z})}{\partial t} + \frac{\partial (n_2 V_{2z}^2 + n_2 \frac{\kappa T_c}{m})}{\partial z} = -\nu_i \, n_2 V_{2z} - \nu_{nw} n_2 V_{2z} \, . \tag{6}$$

The exact stationary density and velocity profiles are compared to those obtained with this macroscopic model on Fig. 2.

CONCLUSION

A simple model for the dynamic of neutrals in a SPT has been derived, taking into account kinetic effects induced by ionization as well as the friction due to the roughness of the walls. The model has been tested in the stationary case by comparison with a 3D Monte-Carlo model. A time-dependent test problem (not reported here) has also shown that the assumptions which could potentially be challenged in time-dependent problems were not critical, thus demonstrating the ability of the model to be used within the global transient model of discharge.

References

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