ARCHING IN GRANULAR MEDIA

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Summary Arching in sand piles can cause a counterintuitive stress depression (a 'dip') at the center of a conical or a wedge-shaped pile. However, predictions of the degree to which the granular material will arch are not easily made. A framework of plasticity analysis is used to shed some light on the issue of arching in granular media.

EXPERIMENTAL EVIDENCE FOR ARCHING IN SAND PILES

The early tests on conical and prismatic piles of sand conducted by Hummel and Finnan [1] revealed very distinct pressure 'dips' under sand piles of both geometries. They also made an observation that clearly links the depression in the base stress to arching: "When the sand was removed at the end of the experiment, it was found that about one-third the height of the cone could be taken away without altering the reading of the centre gauge."

Experiments on conical piles of sand and granular fertilizer by Smid and Novosad [2] confirmed the presence of a stress depression under the center of a conical mass. The heaps were prepared by pouring the granular material through a funnel over a steel platform.

A set of recent laboratory measurements of stress distributions by Vanel et al. [3] beneath both the conical and prismatic heaps indicated the influence of the construction process (or history of sand deposition) on the formation of a pressure 'dip'. The heaps formed by pouring the sand from a point source (conical piles) or a line source (prismatic piles) exhibited a dip, whereas the piles produced by uniform 'raining' did not.

MODELING

Efforts toward description of the distribution of stress in granular piles include both the discrete and continuum approaches (e.g., [4,5] and for a comprehensive review see [6]). The authors' interest in the subject is primarily in the context of the limit state analysis. Solving for stresses under sand heaps is not a typical problem of limit analysis since sand piles are inherently stable, but we are making the argument that the static theorem of limit analysis is useful in assessing fields that promote arching. When this approach is applied to "standard" problems, admissible stress fields are sought to provide a lower bound on an active force causing incipient failure (or an upper bound on a reaction). Although this approach does not guarantee finding the true stress distribution, it shows the correlation of the stress 'dip' and the arching phenomenon.

Radial stress field

We invoke the concept of the plastic radial stress field [7], and we use it as a means of constructing admissible stress distributions in a prismatic sand pile. Irreversible deformation of granular rate-independent materials becomes possible once the plastic stress state is reached. When the incipient failure is of interest (rather than an advanced process of plastic deformation), perfect plasticity is a common assumption, with the Mohr-Coulomb yield condition (f = 0) describing the stress state at failure. Statically admissible stress fields satisfy the stress boundary conditions, are in equilibrium, and do not violate the yield condition. The last requirement indicates that the stress field can be at the limit, as prescribed by: $f = 0$, or the stress state can be below the yielding level: $f < 0$. Therefore, to widen the class of admissible stress fields we introduce a stress function similar to Mohr-Coulomb condition, but with internal friction angle ϕ replaced with variable ϕ^* that is less or equal to ϕ

$$\frac{\sigma_1}{\sigma_3} = \frac{1+\sin\phi^*}{1-\sin\phi^*}, \qquad \phi^* = \phi^*(\theta) \ \le\ \phi \tag{1}$$

We now consider that ϕ^* can be a function of space, and in the special problem of a prismatic pile of sand we assume that ϕ^* is a function of coordinate θ (but not r) of a polar coordinate system with the origin at the apex of the pile. Combining the closure in eq. (1) with differential equations of equilibrium and postulating radial stress field one arrives at the set of two ordinary differential equations for stress distribution

$$\frac{d\chi}{d\theta} = \frac{\cos(2\psi'+\theta) + \chi(\sin 2\psi' + \frac{d\phi^*}{d\theta}\cos\phi^*)}{\cos 2\psi' - \sin\phi^*}, \qquad \frac{d\psi'}{d\theta} = \frac{\sin\theta - \sin\phi^*\sin(2\psi'+\theta) - \chi\cos\phi\,(\cos\phi^* + \frac{d\phi^*}{d\theta}\sin 2\psi')}{2\chi\sin\phi^*(\cos 2\psi' - \sin\phi^*)} - 1 \tag{2}$$

When ϕ^* is constant ($d\phi^*/d\theta = 0$) and equal to ϕ, these equations become identical to classical Sokolovskii radial stress equations.

Arching stress distribution in a prismatic sand pile

Equations (2) were used to calculate the admissible stress state in a sand pile with a distribution of ϕ* governed by an exponential function of θ, where the soil reaches yielding in the 'outer regions' but it is in elastic range in the middle

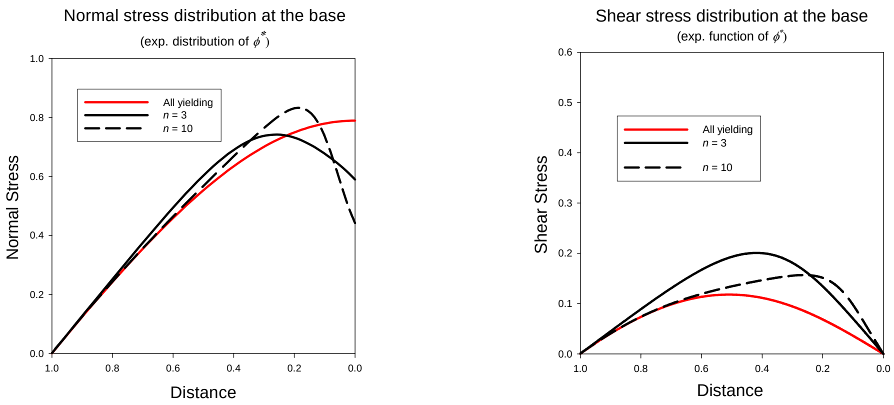

Figure 1. Normalized stress under wedge-shaped sand pile: (a) normal, and (b) shear stress.

part of the pile. The distribution of the base stress underneath the sand pile is illustrated in Fig. 1, along with a distribution for an all-in-failure assumption. Clearly, when the sand is all at yielding, no stress 'dip' occurs, whereas a clear depression is seen when the stress is in the elastic state in the core of the pile.

CONCLUSIONS

Construction of statically admissible stress fields is a standard method in limit analysis for finding a lower bound to an active limit load, or an upper bound to a reaction. Here, the method was modified to indicate the tendency of the stress distribution within the structure to arching. A pile of sand reaches the state closest to failure (consistent with a "spreading" mechanism) when the horizontal component of the reaction under a symmetric half of the pile reaches its minimum. Of different radial stress fields considered, the one with fully plastic stress appeared to yield the minimum of the shear force at the base. Once the stress fields were modified to include below-yielding regions, the horizontal reaction increased, and arching stress fields were found, distinguished by a characteristic 'dip' in the distribution of the normal stress at the base.

The theorems of limit analysis can be rephrased to assess the likelihood of arching occurrence. The static theorem then indicates that *arching may occur and failure will not take place if a statically admissible stress field supporting arching can be found*. Of course, proving that an admissible arching stress field exists does not exclude "non-arching" stress fields from occurring. The second theorem then states that *arching will not occur and collapse is imminent if a kinematically admissible mechanism can be found where the work rate of external loads exceeds the rate of internal work.*

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