Evolution of internal structure of sheared dense granular flows: crystallization transition and non-unique final states

J.-C. Tsai and J. P. Gollub
Physics Department, Haverford College, Haverford, PA 19041, USA
Physics Department, Univ. of Pennsylvania, Philadelphia, PA 19104, USA

It is known that uniformly sheared particle suspensions often exhibit ordering that leads to significant rheological changes [1-3]. Can a sheared dense granular flow, whose velocity profile is highly non-uniform, also exhibit spatial order? How does the internal order influence the flow profiles and the shear forces? And how does the shearing process influence the state of internal order? Previous investigations of granular shear flow have sometimes noted layering of particles in the interior [4]. However, the consequences of internal order for the rheology of sheared granular flows have not been fully explored.

In our initial work on this problem [5], we discovered that shearing can trigger a crystallization transition accompanied by a step compaction event. The delay preceding the transition depends strongly on the layer thickness and can require a translation of about 10^5 particle diameters. The internal velocity fields are qualitatively altered by this transition.

In further work to be reported here, we use the simultaneous measurement of shear force, volume changes and internal imaging to elucidate the connection between internal order and rheology. We also investigate the role of boundary conditions and the shearing protocol in the ordering process. Several striking features are found. The shear force declines significantly as a consequence of the crystallization. Furthermore, the transition can be influenced by the boundary conditions imposed even far from the shearing surface. Most importantly, sheared flow can exhibit non-unique final states; the stability of the disordered state depends on prior shearing history.

APPARATUS (Fig.1)

Spherical glass beads of diameter d = 0.6mm fill the annular channel formed by two concentric smooth stationary cylinders. Glass beads are driven by the floating upper boundary, which rotates at a constant speed while exerting a fixed normal load on the beads; the contact surface is roughened by a glued mono-layer of 0.6mm glass beads. Three different lower boundaries are available: (1) flat bottom (2) mono-layer bottom, and (3) bumpy bottom – a mixture of 0.6mm and 1.0mm beads. The inter-particle space is filled with fluorescent fluid whose refractive index matches that of the glass beads; when illuminated by a laser sheet, internal image slices can be created and captured by CCD cameras. The gap between the driving upper surface and the sidewalls is less than 0.5d; this allows the fluid to enter and exit freely while the particles are confined. The vertical displacement of the upper surface indicates the change of granular volume. Meanwhile, the boundary shear force can be determined by measuring the torque exerted by the driving motor.

CRYSTALLIZATION TRANSITION (Fig.2,3)

Long-term shearing can lead to a sharp crystallization transition. This is shown in Fig.2(a): images of a vertical slice along the center-line of the channel before and after the transition, with a Fourier measure of spatial periodicity showing a step rise at the transition time. The image on the right clearly exhibits a 24-layer structure.

In addition to structural change, the granular volume and shear force decline significantly [Fig.2(b,c)] at the time of the transition. Tracking the speed of particles in the lower region of the flow (marked as Γ in the upper right image) indicates that particles here translate more slowly after crystallization [Fig.2(d)]. The velocity profile (Fig.3) shows quantitatively that particle speed decays more rapidly with depth in the crystallized state. These facts suggest that in the crystallized state, downward transfer of horizontal momentum is less effective than in the disordered state, even though the crystallized state is denser.

Effect of the bottom boundary condition – The long-term shearing induces a sharp ordering transition in the case of flat or mono-layer bottom. The use of a bumpy lower boundary can inhibit the crystallization everywhere. When pure unidirectional shear is applied, the system consistently selects a disordered final state.

NON-UNIQUE FINAL STATES (Fig.4)

If a few cycles of oscillatory shear are applied prior to the long-term shearing, the route of the evolution of a disordered packing (with a bumpy bottom) shows stochastic features. The system can *either* crystallize *or* approach a final state whose interior remains disordered; in this case, specifying the boundary condition and shearing protocol does not uniquely determine the outcome of the evolution.

History-dependent stability of the disordered state – The disordered state, after being sufficient compacted by long-term unidirectional shearing, is found to become stable against oscillatory boundary driving (previous paragraph).

CONCLUSIONS

The evolution of a continuously sheared dense granular packing can be non-unique. Compared to a disordered flow, a crystallized flow requires less shear force and shows a more rapid decline of grain speed with depth. Boundary conditions can significantly influence the spontaneous crystallization under long-term shearing. Meanwhile, a theoretical explanation of the history-dependent stability of the disordered state is an open question.

This work is supported by the US-NSF under grants DMR-0072203 and DMR-0079909.

References

[1] Haw M.D. et al. Colloidal Glasses under Shear Strain. *Phys.Rev. E* **58**: 4673-4682, 1998.

[2] Paulin S, Ackerson B.,Wolfe M.: Microstructure-Dependent Viscosity in Concentrated Suspensions of Soft Spheres. *Phys.Rev.E* **55**:5812-19, 1997.

[3] Sierou A, Brady J: Rheology and Microstructure in Concentrated Noncolloidal Suspensions. *J. Rheol.*,**46**:1031-56, 2002.

[4] Mueth D et al. Signatures of Granular Microstructure in Dense Shear Flows. *Nature*, **406**: 385-9, 2000.

[5] Tsai J.-C., Voth G.A., Gollub J.P.: Internal Granular Dynamics, Shear-Induced Crystallization, and Compaction Steps. *Phys.Rev.Lett.***91**: 064301,2003.

Figures

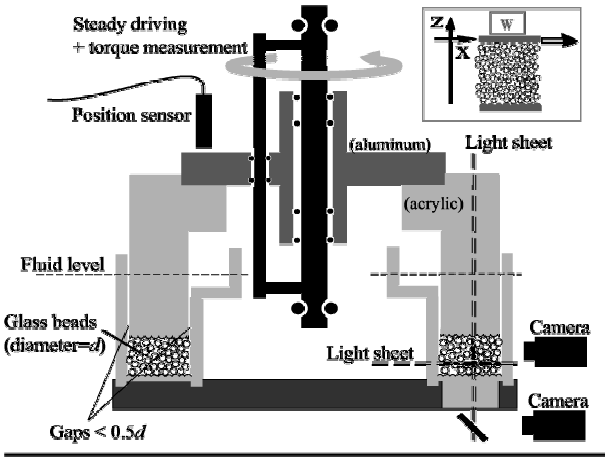

Fig. 1. Cross-section of the annular plane shear flow. The shear and normal load are provided by a rotating aluminum-acrylic assembly that is free to move vertically while maintaining constant rotation rate. The channel width is about 30*d* (*d*=bead diameter) and its circumference is about 800*d*. (The weight of the upper boundary assembly provides a constant normal load, which is more than ten times the total weight of the grains at the typical filling height of 24*d*.)

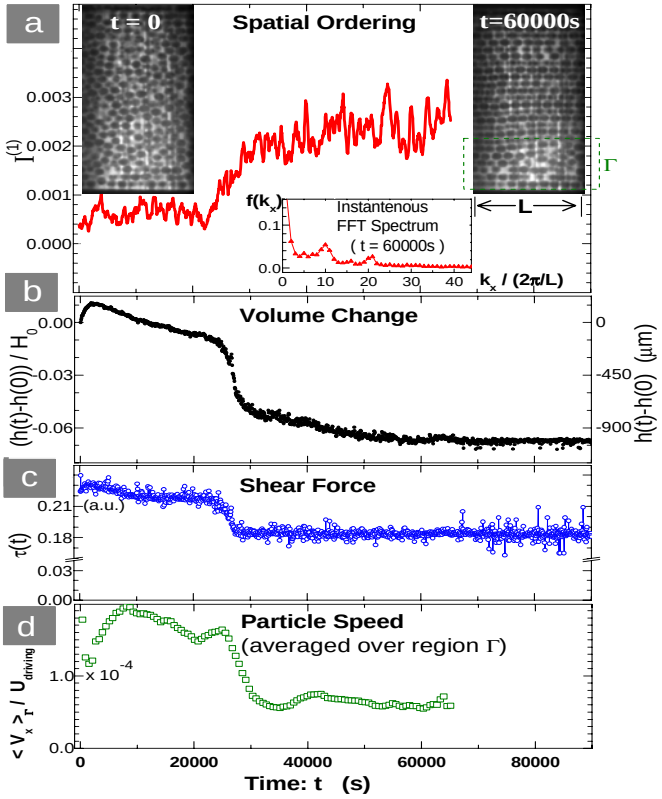

Fig.2. (color) Simultaneous measurement of internal ordering, volume change, shear force, and particle speed, as functions of time. Glass beads are driven at a speed of 12*d*/s from above. (a) Vertical image slices before and after the crystallization, with $I^{(1)}$ showing the Fourier intensity corresponding to a spatial period of 1d in the horizontal direction. (b) Fractional change of total height, as an indicator of total volume change. (c). Shear force (d) Particle speed averaged over the lower region Γ, marked on the upper right image.

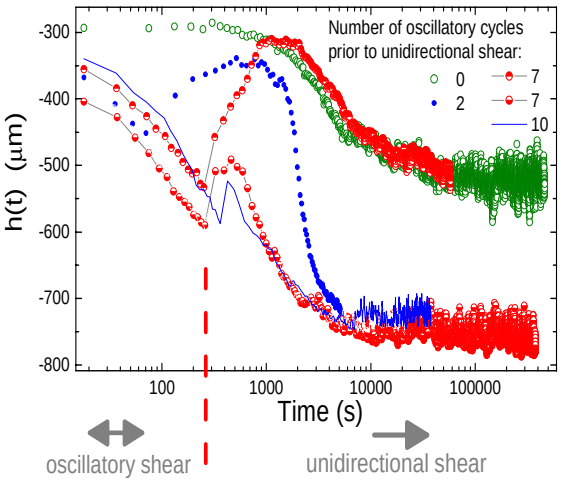

Fig. 4. (color) The height h(t) indicates the final state of internal ordering. With pure unidirectional shearing (labeled empty circles), the system always approaches a disordered final state (joining curves at the top right). On the other hand, oscillatory shearing for a few cycles causes the system to evolve stochastically into either a disordered state, or a denser crystallized state (curves at the bottom right).

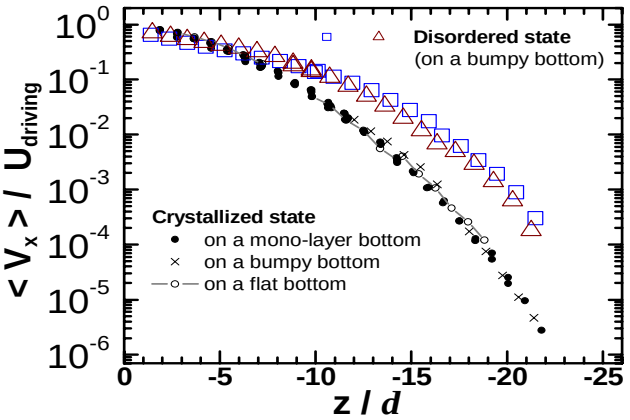

Fig. 3. (color) Comparison of the asymptotic mean velocity profiles for different final states. (The triangles and squares represent velocity profiles measured at different vertical planes of the disordered state.)