THE EFFECT OF DIFFERENT PARTICLE CONTACTS ON SUSPENSION RHEOLOGY

Helen J. Wilson*, Robert H. Davis**
*Department of Applied Mathematics, University of Leeds, Leeds, UK
**Department of Chemical and Biological Engineering, University of Colorado, Boulder, CO 80309-0424 USA

Summary In this paper we consider various different models of inter-particle contact in low Reynolds number flow to illustrate the effect of contact on suspension rheology. Experiments have shown that contact between a spherical particle and an inclined plane occur at an effective roughness height that depends on the angle of inclination of the plane. We use two different methods to model this behaviour, and use numerical simulation to investigate the dependence of the suspension properties on the particle contacts.

INTRODUCTION

Many important processes, both natural and technological, involve flowing suspensions and small solid particles transported in a Newtonian liquid. An understanding of the effect of the solid particles on the macroscopic stress of the flowing fluid is critical for applications such as separating solids from liquids by settling, cleaning sediment from pipes by viscous resuspension, and slurry flows.
Recent studies (e.g. [2, 8, 9]) have shown that the small-scale interactions between particles in suspension can have a large effect on the rheology of the macro-scale (apparently homogeneous) fluid. A particular effect which has only recently been considered is that of particle roughness. Perfectly smooth particles without interparticle forces in an inertialess flow should never make contact, and their interactions should be entirely reversible. However, experiments [7] have shown that reversible interactions do occur, and various models of this *contact* interaction have been proposed. In this paper we consider existing and new models of the contact interaction and investigate the effect of contact on suspension rheology, and also the extent to which our results depend on the contact model used.

CONTACT MODELS

Benchmark model

The first model of interparticle contact was introduced by Davis [4] and Hinch [3]: both these papers introduced models which have, as a special case, frictionless roll-slip contact. This is also a special case in models used elsewhere [2, 5]. Particles are assumed to have asperities which do not affect their hydrodynamic interaction. These asperities cause apparent contact between particles when their surfaces reach a specific nominal separation. The contact prevents further approach of the particles, but a contact force can only resist compression, so when the flow acts to separate the particles the roughness ceases to have any effect. This simple one-parameter model (the parameter being ξ, the dimensionless nominal surface separation at contact) is the start-point for most studies of contact effects. Experiments by Smart & Leighton [7] suggest that an appropriate physical range for the dimensionless roughness height is $0.001 \leq \xi \leq 0.01$.

Recent Experimental Observations

The driving force behind the new contact models discussed here is a series of experiments carried out by Zhao, Galvin & Davis [10]. In these experiments, a single sphere travelled down an inclined plane in a viscous fluid at very small Reynolds number. When the apparatus was inverted, the time for the sphere to move away from the plane was measured and used to deduce the sphere's initial separation from the plane. They found that the steeper the angle of inclination of the plane, the larger the measured displacement between sphere and plane during quasi-steady motion down the incline.

First paradigm: multiple roughness heights

The model devised by Zhao *et al.* to reproduce the behaviour described above is based on the idea of sparse asperities with two different effective roughness heights. The fundamental idea behind the model is that a sphere travelling down an inclined plane spends some time in contact with the plane through its large asperities, some in contact through its small asperities and some time falling towards the surface after contact with a large asperity. For a shallow incline, the falling motion is almost directly towards the plane, so the time spent far from the plane is low; for a steep incline the motion after contact with a large asperity is nearly parallel with the plane, so the sphere spends most of its time at a distance from the plane corresponding to the size of the large asperities.
The model has three parameters: the size of the small asperities, the size of the large asperities, and an angle parameter determining the typical spacing between large asperities. A possible generalisation of this model is to consider a Gaussian distribution of asperity heights: again three parameters will suffice, the mean and variance of the distribution and number of asperities per particle. This new model is only accessible via numerical simulation, but is likely to be more realistic than the two-height model.

Second paradigm: compressible asperities

We propose a new way to model the experimental observations above. We treat the asperities as a soft *roughness layer*, which deforms under compression. Thus a sphere on a shallow incline feels a large force (from the normal component of its weight) normal to the incline, which compresses the asperities and yields a low roughness height and large contact force; on a steep incline, the component of gravity normal to the incline is small, so there is little compressive force and a small contact force is sufficient to halt the sphere's approach to the incline, so the roughness layer remains larger.

In order to obtain the simplest possible model that avoids very close approach, we simply treat the compressible layer as a spring, with responds only to compression to under its natural length with a simple nonlinear spring force $\underline{F}$:

$$\underline{F} = \lambda \frac{(\xi - h)}{h\xi} \underline{n} \quad \text{if} \quad h \leq \xi, \qquad \underline{F} = \underline{0} \quad \text{otherwise,}$$

where h is the nominal separation of the sphere surfaces, and ξ and λ are the two spring parameters. The contact force is continuous at $h = \xi$ and diverges as the separation is reduced to zero.

NUMERICAL METHOD

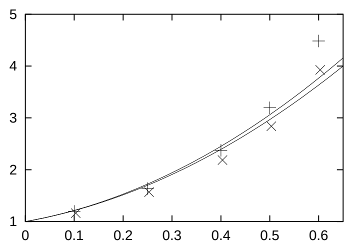
We simulate shear flow of a suspension of solid spheres using a modification of the Stokesian Dynamics (SD) method. In this method, pioneered by Brady & coworkers [1], far-field contributions to the mobility formulation are calculated first, then inverted to calculated a far-field resistance matrix which correctly includes screening; pairwise lubrication interactions are then added directly to the resistance matrix. We use the Accelerated Stokesian Dynamics method [6], which reduces the complexity in return for neglect of lubrication interactions between well-separated particles. In addition, we carry out some simulations in which we neglect far-field interactions completely (which is not too bad an approximation for large dense systems). This reduces the complexity to $O(N)$, where N is the number of particles per periodic box, allowing us to carry out a few calculations at very high concentrations and for very large numbers of particles and to assess the importance of both far-field interactions and box-size. Three-dimensional simulation allows us to calculate the three key rheological quantities of shear viscosity, first and second normal stress differences.

In order to allow for contact forces, we split the time-evolution into two stages. First we calculate the velocities of all spheres in the absence of contact. Then we use this information to approximate the contact forces for each pair of particles, using lubrication theory. Next the particle velocities are calculated numerically with the contact forces acting. In the case of incompressible asperities, the process is iterated to convergence, calculating small corrections to the contact forces.

PRELIMINARY RESULTS AND CONCLUSIONS

Initial two-dimensional studies have been carried out using the original contact model [4] and the full Stokesian Dynamics method. The graph on the right shows the dependence of shear viscosity on area concentration of solids for two different roughness heights, indicating that an increase in ξ decreases the shear viscosity. The curves are theoretical viscosity values from [9], valid for dilute concentrations under certain assumptions. These also predict the trend that increasing contact height will lower the suspension's shear viscosity. The reason for this is that the asperities prevent close approach of the particles, thus limiting the dissipation in lubrication layers between particles.

These preliminary studies also indicate that particle contacts cause a small negative first normal stress difference. We will present further results on the variation of this value with contact height, and on second normal stress differences (for which three-dimensional numerical studies are required).



Suspension viscosity, made dimensionless by fluid viscosity, plotted against particle area fraction. The roughness heights are $\xi = 10^{-3}$ (+) and $\xi = 10^{-2}$ ($\times$).

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