SEDIMENTATION OF DILUTE SUSPENSIONS

B.U. Felderhof
Institut für Theoretische Physik A, RWTH Aachen, Templergraben 55, 52056 Aachen, Germany

Summary For a class of sedimentation states of a dilute suspension, corresponding to definite values of the mean sedimentation velocity and of the horizontal and vertical velocity variance, the static structure factor and the velocity correlation functions of the suspended particles are calculated and shown to exhibit screening.

The sedimentation of particles in a viscous fluid is still not fully understood, even for settling at low Reynolds number. Batchelor[1] first showed that for disordered suspensions, with complete neglect of correlations in particle positions apart from hard-sphere repulsion, the sedimentation velocity decreases rapidly with volume fraction ϕ, in proportion to $1 - 6.55\ \phi$. However, in his calculation he assumed that the pair distribution of the suspended particles is isotropic, and corresponds to that of a dilute hard-sphere system. The determination of the actual statistical distribution of a sedimenting suspension and the calculation of the corresponding sedimentation velocity is still an open problem.
It is known since the work of Caflisch and Luke[2] that for dilute suspensions, where the point approximation is reliable, the variance of particle velocity diverges with the size of the system if the distribution is random. It has been suggested that a form of hydrodynamic screening is necessary to keep the variance finite[3]. The divergence with the size of the system is not seen in experiment[4][5][6]. Several theoretical explanations have been advanced[7][8] that do not rely on the screening property. Our calculation explores the effect of particle correlations due to hydrodynamic interactions.
Hydrodynamic interactions are simple for dilute suspensions, since then the particles are far apart and a point approximation can be used. We have shown that for sedimentation states that are characterized by a distribution in configuration space of simple form, the horizontal and vertical variance of velocity can be calculated by use of a continuum approximation. The variances tend to finite values in the limit of an infinite system. We have also calculated the corresponding velocity correlation functions, density correlation function, and structure factor. The correlation functions show screening due to an interplay of Boltzmann entropy and hydrodynamic interactions. Parameters of the state can be chosen to obtain agreement with experiment. If the parameters are chosen to be independent of density, then the correlation functions automatically show the scaling with density seen in experiment[6].
Due to the anisotropy of the pair distribution Batchelor's low density result for the friction coefficient must be corrected. We find that to lowest order in density the correction to the Stokes velocity is given by

$$\bar{u}_{z0} = \frac{K}{6\pi\eta a} - \mu_0 \frac{K}{6\pi\eta d} \tag{1}$$

with numerical coefficient $\mu_0 \approx 0.028$. Here K is the force acting on a particle, η is the shear viscosity, a is the particle radius, and $d = (3/4\pi n_0)^{1/3}$ is the mean distance between particles at number density n_0. The velocity variances in the dilute limit are

$$\Delta V_{||0} = \alpha_z \frac{K}{6\pi\eta d}, \qquad \Delta V_{\perp 0} = \alpha_h \frac{K}{6\pi\eta d}, \tag{2}$$

with numerical coefficients $\alpha_z \approx 2.1$ and $\alpha_h \approx 0.9$, as found by comparison with experiment[6]. The quantity $v_K = K/(6\pi\eta d)$ is the velocity scale of the dilute suspension.

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