VELOCITY FLUCTUATIONS IN NON-BROWNIAN SUSPENSIONS UNDERGOING SIMPLE SHEAR FLOWS

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Summary The velocity fluctuations present in macroscopically homogeneous suspensions of neutrally buoyant, non-Brownian spheres undergoing simple shear flow, and their dependence on the microstructure developed by the suspensions, are investigated in the limit of vanishingly small Reynolds numbers using Stokesian dynamics simulations. We show that, in the dilute limit, the standard deviation of the velocity fluctuations (the so-called suspension temperature) is proportional to the volume fraction, in both the transverse and the flow directions, and that a theoretical prediction, which considers only the hydrodynamic interactions between isolated pairs of spheres, is in good agreement with the numerical results even up to concentrations $\phi \sim 10\%$.

SUSPENTION TEMPERATURE

The problem of determining the velocity fluctuations in suspensions of non-Brownian solid spheres in Stokes flows is one of long-standing difficulty due to the underlying long-range many-body hydrodynamic interactions between the suspended particles. The variance, or the standard deviation (STD), of the velocity fluctuations is the simplest measure of the magnitude of such fluctuations and is sometimes referred to as the suspension *temperature*, which in the case of an anisotropic motion of the suspended spheres would actually be a tensor (covariance matrix). It is the purpose of this paper to investigate the velocity fluctuations present in a macroscopically homogeneous, unbounded suspension of neutrally buoyant, non-Brownian spheres subject to a simple shear flow in the limit of vanishingly small Reynolds numbers using Stokesian dynamics, as well as their dependence on the suspension microstructure generated by the shear.

The simulations include a non-hydrodynamic repulsive force between the spheres which, although extremely short ranged, leads to the development of fore-aft asymmetric distributions for large enough volume fractions, if the range of that force is kept unchanged. On the other hand, we show that, although the pair distribution function recovers its fore-aft symmetry in dilute suspensions, it remains anisotropic and that this anisotropy can be accurately described by assuming the complete absence of any permanent doublets of spheres.

	$t_{11} = t_{22}$	t_{33}		$t_{11} = t_{22}$	t_{33}
Random Hard Sphere ($g_{\text{HS}}(r)$)	0.3157	0.0811	Simple Shear Flow ($g_{\text{BG}}(r)$)	0.4637	0.1031

Table 1. Temperature tensor in the dilute limit, computed using Eq. 1 for two different pair distribution functions, one for a hard-sphere distribution, and the other given by [1] for a simple shear flow in the dilute limit.

In the dilute limit, the fluctuations in the velocity come from two-particle interactions, and from the far-field form of these interactions it can be shown that any component of the temperature tensor, of the form $\delta v_i \delta v_j$, decays faster than $1/r^3$ and therefore, its average value can be directly computed by averaging the hydrodynamic interaction between a pair of spheres over all possible configurations,

$$T_{ij} = \int d\mathbf{r}\ \delta v_i \delta v_j \ (3\phi/4\pi)\, g(\boldsymbol{r}) = \phi \left[\frac{3}{4\pi} \int d\mathbf{r}\ \delta v_i \delta v_j g(\boldsymbol{r}) \right] = \phi\, t_{ij}, \tag{1}$$

which gives a linear dependence of the temperature components on the volume fraction, $T_{ij} = \phi\, t_{ij}$.

The exact temperature values will depend in general on the pair distribution function. In Table 1, we present the diagonal terms of the temperature tensor in the dilute limit, obtained from the numerical integration of Eq. 1 for two different isotropic pair distribution functions, corresponding to a random distribution of hard spheres, $g_{\text{HS}}(r) = 1$, and to Batchelor & Green's result for a suspension in a simple shear flow, $g_{\text{BG}}(r)$ [1].

In figure 1, we present the diagonal terms of the temperature tensor as a function of the volume fraction, obtained in a simple shear flow by means of Stokesian dynamics simulations [2]. The temperature components T_{11} and T_{22} converge to a common curve in the dilute limit, which is consistent with the existence of an isotropic pair distribution function and indicates that the effect of the particle-depleted region of closed streamlines is not measurable.

In addition, the decay of the velocity fluctuations follows the dilute limit scaling given by Eq. 1, viz. that T_{ij} is proportional to ϕ, even for surprisingly high volume fractions.

On the other hand, at larger concentrations we see that the T_{11} and T_{22} curves separate from each other, which is evidence of the structure developed by the suspension at high concentrations. In fact, in a recent paper we showed that, although we have a very short-ranged interparticle force, $r_c = 10^{-4}$, and that the pair distribution function has fore-aft symmetry in the dilute limit, for larger concentrations this symmetry no longer holds [3].

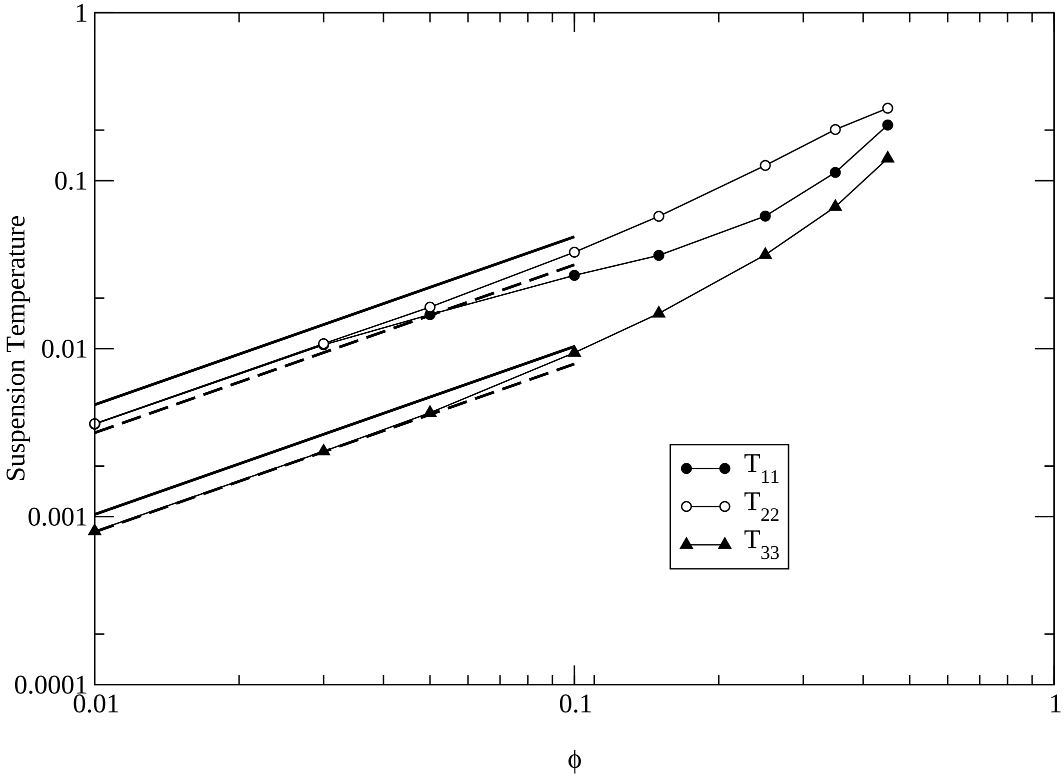

Figure 1. Diagonal components of the temperature tensor as a function of the volume fraction, obtained from the numerical Stokesian dynamics simulations. The solid lines correspond to the dilute limit, i.e. two-sphere, calculation using $g_{\text{BG}}(r)$ for $T_{11;22} = \phi\, t_{11;22}$ and $T_{33} = \phi\, t_{33}$, respectively. The computed values of $t_{11;22}$ and t_{33} are given in the second part of table 1 and the discrepancy between the dilute limit theory and the Stokesian dynamics results is about 25% for $\phi < 0.1$. The dashed lines correspond to the dilute limit calculation for $T_{11;22} = \phi\, t_{11;22}$ and $T_{33} = \phi\, t_{33}$, respectively, assuming a random distribution of hard spheres, i.e. $g_{\text{HS}}(r) = 1$. The computed values of $t_{11;22}$ and t_{33} are given in the first part of table 1 and the discrepancy with the Stokesian dynamics results is about 5%, again for $\phi < 0.1$.

We also show in figure 1 that, the calculations using $g_{\text{HS}}(r) = 1$, provide a surprisingly good estimate to the velocity fluctuations in sheared suspensions, suggesting that, probably, the effect of the microstructure is not measurable in the dilute limit.

This surprising result may be due to the fact that the main difference between the temperature, as obtained from the two-sphere calculations, using either $g_{\text{BG}}(r)$ or $g_{\text{HS}}(r)$, originates in the contribution of the lubrication region, which is negligible in the $g_{\text{HS}}(r) = 1$ case (see table 1).

Thus, a lower bound to the velocity fluctuations in the absence of permanent doublets can be estimated roughly using $g_{\text{HS}}(r) = 1$. Also, due to the slow, $\mathcal{O}(1/r^2)$, decay of the velocity fluctuations with the distance between spheres, the contribution of the lubrication region to the temperature is relatively minor, $\sim 20\%$, for a pair distribution function given by $g_{\text{BG}}(r)$, and should be expected to become even smaller due to the depletion of permanent doublets. However, the observed excellent agreement between the Stokesian dynamics results for the temperature of sheared suspensions and the temperature obtained from the two-sphere calculations using $g_{\text{HS}}(r) = 1$ could not have been expected a priori and is a surprising result.

This work was supported by the Engineering Research Program, Office of Basic Energy Sciences, U.S. Department of Energy under Grant DE-FG02-03ER46068.

References

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