HYDRODYNAMIC THEORY OF DE-WETTING

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Summary A prototypical problem in the study of wetting phenomena is that of a solid plunging into or being withdrawn from a liquid bath. In the latter, de-wetting case, a critical speed exists above which a stationary contact line is no longer sustainable and a liquid film is being deposited on the solid. Demonstrating this behavior to be a hydrodynamic instability close to the contact line, we provide the first theoretical explanation of a classical prediction due to Derjaguin and Levi: instability occurs when the outer, static meniscus approaches the shape corresponding to a perfectly wetting fluid.

INTRODUCTION AND RESULTS

The forced wetting or de-wetting of a solid is an important feature of many environmental and industrial flows. In typical applications such as painting, coating, or oil recovery it is crucial to know whether the solid will be covered by a macroscopic film or not. If a fiber is plunging into a liquid bath to be coated (wetting case), the speed can be quite high (m/sec), while maintaining a stationary contact line. In the opposite case of withdrawal (de-wetting), a stationary contact line is observed only for very low speeds, and a macroscopic film is deposited typically at a speed of only a few cm/sec. Yet no theoretical explanation for this instability exists, or of the fundamental difference between the two cases.

It is well known that viscous forces become very large near a moving contact line, and are controlled only by some microscopic cut-off λ, for example a slip length. As a result of the interplay between viscous and surface tension forces, the interface is highly curved, and the contact line speed U is properly measured by the capillary number $Ca = U\eta/\gamma$, where η is the viscosity of the fluid and γ the surface tension between fluid and gas. It was first proposed by Derjaguin and Levi (1), that instability occurs if this dynamic interface angle reaches *zero*. Apart from the fact that there is no justification for this condition, it does not lead to a unique criterion since the angle depends on the position where it is evaluated. However, it was noted experimentally (2) that the interface profile at the critical speed corresponds to a *static* meniscus with zero equilibrium contact angle, except in the immediate neighborhood of the contact line.

To explain the physical mechanism behind the instability, we note that the interface shape $h(x)$ near the contact line must have the form

$$h(x) = 3\lambda H(\xi), \quad \xi = x\theta_e/(3\lambda), \tag{1}$$

since the cut-off λ is the only available length scale. The dependence on θ_e was put in for later convenience. As expected, the curvature $h''(0) = \theta_e^2/(3\lambda)H''(0)$ becomes very large at the contact line, since λ is in the order of nanometers. As the curvature of the local solution has to match to a value of order unity in the static outer part of the profile, the usual boundary condition for H is one of vanishing curvature for large ξ. In the wetting case this leads to an asymptotic solution first found by Voinov and an expression for the interface angle usually referred to as "Tanner's law".

However, it is a well-known but little appreciated fact that Voinov's solution fails away from the contact line in the de-wetting situation. Instead, the local solution always retains a positive curvature, and will fail to match to an outer solution in the limit of small λ. At the critical speed, the necessary compromise between the inner solution near the contact line and the outer static solution has been pushed to the limit: from all possible inner solutions, the one with the *smallest* possible curvature is selected. The outer solution, on the other hand, realizes the solution with the *largest* curvature, which happens to be the one corresponding to zero contact angle, in agreement with the criterion of Derjaguin and Levi.

To make an explicit comparison between our asymptotic theory and numerical integration of the lubrication equations, we choose a plate being withdrawn from a liquid bath (see Fig.1) as a test case, a geometry which was studied in detail in (3). There it was found numerically that there exists a critical capillary number Ca_{cr}, above which no static solution exists, in agreement with the experimental findings described above. The final result of our theory is then

$$\delta_{cr}^{1/3} \exp[-1/(3\delta_{cr})] = 2^{1/3}\pi(Ai(s_{max}))^2\mu\theta/\theta_e, \tag{2}$$

where $\delta_{cr} = 3Ca_{cr}/\theta_e^3$ is the critical rescaled capillary number, and $\mu = \lambda/\theta_e$ the rescaled slip length.

Our central result (2) is compared in Fig.2 to a numerical solution of the lubrication eqaution. No parameter was adjusted to achieve this comparison.

References

[1] B.V. Derjaguin and S.M. Levi, *Film coating theory*, Focal Press, London (1964).

[2] R.V. Sedev and J.G. Petrov, Colloids and Surfaces, **53**, 147-156 (1991).

[3] L.M. Hocking, Euro. J. Appl. Math. **12**, 195-208 (2001).

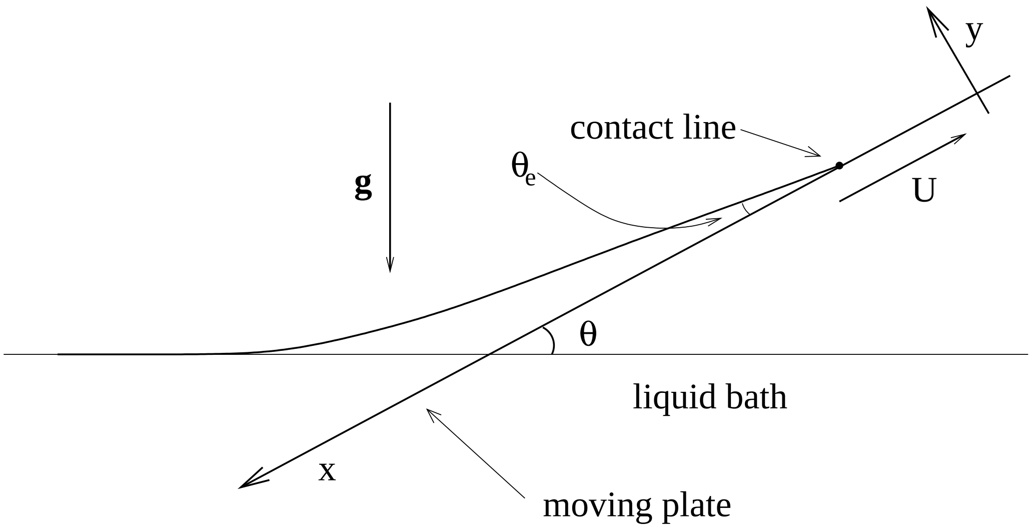

Figure 1. A schematic of the setup. At the contact line, h(0) = 0, and the slope of the interface is θ_e. The plate is being withdrawn at an angle θ.

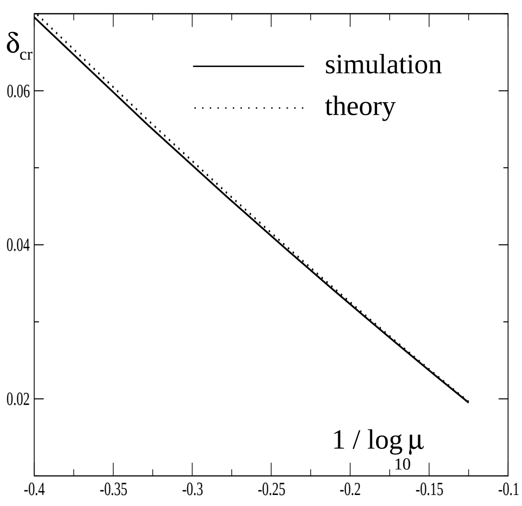

Figure 2. A comparison of δ_{cr} as determined by numerical integration of the lubrication eqaution for $\theta/\theta_e = 1$, and theory, as summarized by equation (2).