A GENERIC MECHANISM FOR BY-PASS TRANSITION IN VORTICES ?

Arnaud Antkowiak* and Pierre Brancher*
*Institut de Mécanique des Fluides de Toulouse, Allée du Professeur Camille Soula, 31400 Toulouse, France

Summary Investigation of the energy amplification potential for an axisymmetric vortex is carried out (*optimal perturbation analysis*). Intense amplifications of perturbations are found to occur for various azimuthal wavenumbers m via an original scenario involving both Orr mechanism and vortex induction. Moreover, $m = 1$ optimal perturbation appears to contaminate the core with the formation of a strong bending wave. $m > 1$ optimal perturbations are also capable of core contamination with the creation of a multipolar strain field. This induced field can potentially be the seed for secondary instabilities (by-pass transition). Finally, the ingredients feeding the growth are present in all vortices, revealing the generic nature of the mechanism.

INTRODUCTION

Generic components of a wide diversity of flows, ranging from industrial to geophysical scales, vortices have been the subject of numerous studies since years. Along these, it appeared that without additional ingredients such as an external strain field or the presence of an axial flow, vortices can hardly encounter instabilities. This strong robustness featuring the vortex is well illustrated by its presence and persistence in so much flows, *e.g.* the hazardous wake of an aircraft.

Some theoretical as well as numerical works have switched off these additional effects in order to focus on the dynamics of an isolated vortex (axisymmetric monopole). Particularly, the process of axisymmetrization has been well documented, and it is now well known that an initial random disturbance injected in a vortex will see its own vorticity gradients steepen thanks to the action of differential rotation until the viscous scale is reached, which means the dying of the perturbation. This is the so-called *shear-diffusion* process acting on an $\mathcal{O}(\mathrm{Re}^{1/3})$ timescale.

Nevertheless, monotonic decay of disturbances, although common, is not systematic. Indeed, clues of non trivial behaviours have been reported with rare but violent cases of disruptions. Moreover, analytical works conducted on hollow vortices (plasma columns and hurricanes) have revealed the possibility for algebraic instabilities to occur.

In this context, our objective is to evaluate the potential for transient energy amplification in an axisymmetric monopole.

FORMULATION

The Lamb-Oseen vortex as an archetype of monopole

The monopole under investigation here will be the classical self-similar solution of Lamb-Oseen, with diffusing vorticity $Z(r,t)$ and angular velocity $\Omega(r,t)$ defined as :

$$Z(r,t) = \frac{2}{1+4t/\mathrm{Re}}\exp\left(-\frac{r^2}{1+4t/\mathrm{Re}}\right)\;;\;\Omega(r,t) = \frac{1-\exp\left(-\frac{r^2}{1+4t/\mathrm{Re}}\right)}{r^2} \tag{1}$$

Here, the initial vortex radius r_0 and initial axis angular velocity Ω_0 have been used to construct the Reynolds number $\mathrm{Re} = \Omega_0 r_0^2/\nu$, where ν denotes the kinematic viscosity.

Non-normality inside

Tracking now the evolution of a disturbance injected in such a flow, we have to solve the governing equations, namely the linearized Navier-Stokes equations, that can be formally written :

$$\mathsf{F}(\boldsymbol{v}) = \mathsf{L}\frac{\partial \boldsymbol{v}}{\partial t} + \mathsf{C}\boldsymbol{v} - \frac{1}{\mathrm{Re}}\mathsf{D}\boldsymbol{v} = \mathbf{0}, \tag{2}$$

where L is a normal operator, D a classical self-adjoint diffusion operator and C is an operator where all the couplings between the mean flow and the disturbance concentrate. It is noteworthy that differential rotation breaks the normality for this operator. This mathematical statement has a direct physical translation : a disturbance can experience a transient stage of energy amplification before conforming to the long-time modal prediction (exponential decay in the present case) [1]. But what quantitative redirection of energy from the mean flow to the disturbance can be expected ? This question is addressed in computing the optimal perturbation.

Optimal disturbance

The optimal perturbation is the initial condition which maximizes the energy growth during a finite time interval. Although different techniques exist, it is convenient to use the optimal control theory to compute it [2]. In this theoretical frame, the problem lies in maximizing the energy growth (the *objective*) at a given time τ under the *constraints* of respecting (2) and the associated boundary conditions. The initial condition is used as a *control* to be adjusted in order to meet the objective. This constrained optimization problem can be solved by considering the equivalent unconstrained problem for a Lagrangian functional, whose stationary point can be found using standard methods.

RESULTS

Azimuthal wavenumber-one optimal perturbation ($m = 1$)

The analysis reveals that considerable growths can be reached for $m = 1$ optimal perturbation, even at moderate Reynolds numbers. This amplification follows from an original scenario [3] involving both a local mechanism (an analogue of the Orr mechanism [4]) and the global vortex induction. Figure 1 illustrates the evolution of this perturbation. As can be noticed, another important fact is the ability of the optimal to contaminate the core of the vortex, although initially located at the periphery. The physical mechanisms feeding the energy growth are basically differential rotation and vortex induction. Therefore the mechanism involved is generic; it does not depend on a particular vorticity profile.

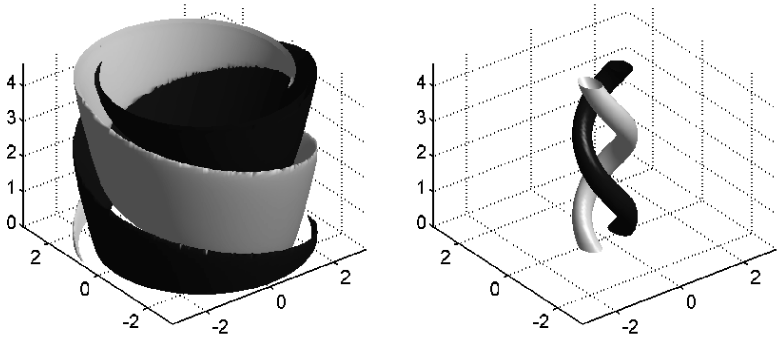

Figure 1. Re = 1000. Initially (left), the optimal perturbation is composed of a set of vorticity sheets lying at the periphery of the vortex. As time flows, the spirals unroll and destructive interference between sheets disappear. Eventually (right), a strong bending wave has been created in the core of the vortex.

Higher wavenumbers optimal perturbations ($m > 1$)

Large growths can still be reached for higher azimuthal wavenumbers (figure 2). Once again, Orr mechanism and vortex induction are associated to the energy amplification. Looking at the evolution of the perturbation, it is found that the main visible difference is due to the role played by vortex induction. For $m = 1$, it results in the creation of a bending wave and for $m > 1$, it produces a transient multipolar strain field in the core of the vortex.

Since secondary instabilities of elliptic kind are known to occur on vortices embedded in an elliptical strain field, it is of interest to look at the nonlinear development of this perturbation. This issue is investigated by means of numerical simulations and preliminary results will be presented, in order to address the relevance of a by-pass transition scenario in vortices.

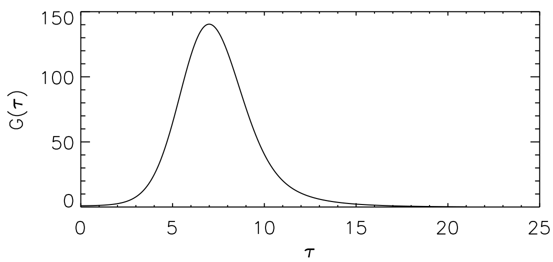

Figure 2. Re = 5000. Typical evolution of the energy growth for a $m = 2$ optimal perturbation.

References

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