# COMPUTATIONAL STUDY OF TURBULENT-LAMINAR BANDS IN COUETTE FLOW

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*Summary*
Direct numerical simulations of plane Couette flow are used to study turbulent-laminar patterns similar to the spiral turbulence state known for many years in Taylor-Couette flow. A technique based on minimal flow unit simulations is employed. Turbulent-laminar patterns are found in close agreement with experiment.

## Introduction

In Taylor-Couette flow between counter-rotating cylinders it is possible to obtain an intriguing state known as spiral turbulence consisting of alternating regions of turbulent and laminar flow [1]. Because turbulent and laminar flow self-organize into well-defined regions with neither invading the other, this state has attracted much attention over the years [1, 2, 3, 4], but has never been explained. Recently, using a Taylor-Couette apparatus with a very narrow gap, Prigent et al. [5, 6] were able to produce a flow containing several regular repetitions of the turbulent-laminar pattern. Furthermore, they discovered that an analogous state exists in plane Couette systems whose gap is very small compared with the lateral dimensions. In both cases, the pattern is oriented obliquely to the streamwise direction (i.e. the azimuthal direction in the Taylor-Couette case) and the wavelength is on the order of 20–30 times the gapsize. Our work is aimed at understanding the onset and structure of the pattern of laminar and turbulent bands in these Couette flows.

## Methods

We use direct numerical simulations (DNS) of plane Couette flow in domains which permit computation of the turbulent-laminar flow at reasonable cost and which allow us to address questions which are not directly accessible to experiment. The simulation domains are illustrated in Fig. 1. Units are chosen such that the plates are separated by a gap of 2 and move at velocities $\pm\mathbf{e_x}$; the Reynolds number is $Re = 1/\nu$. It is known from minimal flow unit (MFU) simulations [7] that a weakly turbulent flow can be sustained in a computational domain with periodic lateral boundaries of size approximately 6 (streamwise) by 4 (spanwise) length units (Fig. 1). The spanwise length is near the minimum streak pair spacing for turbulent Couette flow near $Re = 400$. Here we extend the MFU computations in two ways. First we allow the domain to be tilted with respect to the streamwise direction while still maintaining periodicity of the computational domain and respecting the required streak spacing. This permits us to study patterns which are oblique to the streamwise direction. Secondly, we consider domains which are long in only one direction. Thus we can simulate large length scales and study the effect of angle and domain length on the formation of the turbulent-laminar patterns.

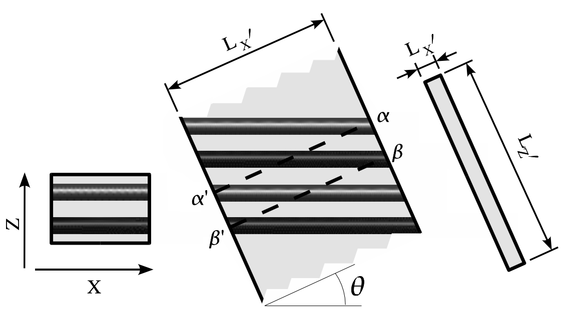
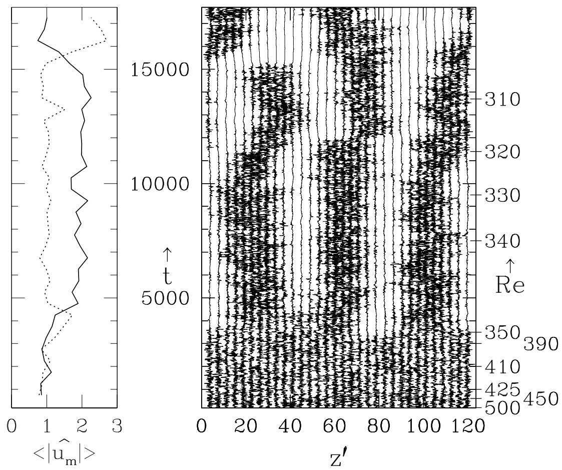



**Figure 1.** Simulation domains. The streamwise and spanwise directions are $x$ and $z$; computational domains are aligned in directions $x'$, $z'$, which are tilted at angle $\theta$. The light and dark gray streamwise bars represent streamwise vortices or streaks with a spacing of 4 length units. Left: MFU domain [7]. Middle: tilted domain at angle $\theta$ with a short length $L_{x'}$ which respects the streak spacing ($L'_x \sin(\theta) \simeq 4$). Right: domains have a large length $L_{z'}$ in one direction. In all cases $L_y = 2$ (not seen).

## Results

Fig. 2 shows the formation of a turbulent-laminar pattern. A turbulent flow is initialized at $Re = 500$. Then $Re$ is decreased, rather quickly, at times indicated. A pattern emerges with three distinct turbulent and laminar regions. This is seen quantitatively in the modulus of the Fourier transform (averaged over 500 time units) by the emergence of a peak at $m = 3$. In the tilted frame the wavelength is 40, agreeing closely with experiment [5, 6]. $Re$ is fixed at 350 long enough to show that the pattern is stable. The flow field is visualized in Fig. 3. Then $Re$ is decreased more slowly. At $Re = 310$ the pattern changes to a two-band structure, seen quantitatively as a rise in the $m = 2$ spectral peak and a decrease in the $m = 3$ peak. Many more simulations have been conducted with different domain sizes and different angles. Fig. 4 shows an isolated turbulent region found at $\theta = 66^o$.



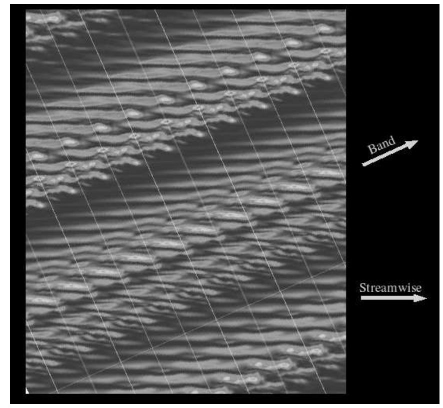




**Figure 2.** Right: Space-time diagram showing formation of the turbulent-laminar pattern. The domain size is $L_{x'} \times L_{z'} = 10 \times 120$ with a tilt of $24^o$. Shown is the spanwise velocity measured at 32 points equally spaced along a line in the middle of the channel ($x' = y' = 0$). Time is shown on the vertical axis with changes in Reynolds number indicated on the right. Left: Time-averaged modulus of the spatial Fourier transform of the data corresponding to modes $m = 3$ (solid) and $m = 2$ (dashed).

**Figure 3.** Visualization of the patterned flow at $Re = 350$. The computational domain (outlined in white) is repeated periodically to tile an extended region. Plotted is the streamwise velocity near the upper wall of the channel. The streamwise streaks, typical of turbulent channel flows, can be seen in the turbulent regions. The pattern is oblique to the streamwise direction.

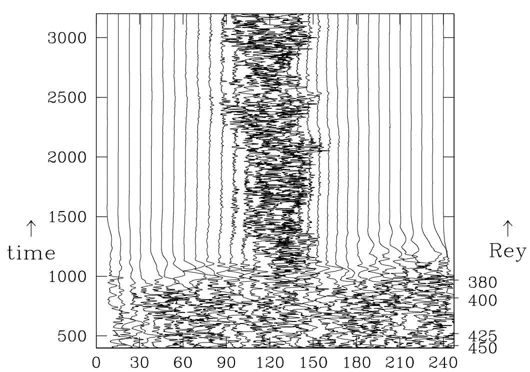



**Figure 4.** Space-time diagram for $\theta = 66^o$. Here the region of turbulent flow appears isolated. It takes up a relatively small proportion of the domain and its size does not change with changes in domain size. Hence there is no selected periodicity to the pattern.

## Conclusions

Our simulations demonstrate that patterns can be generated with only one extended lateral direction ($L_{z\prime}$). The other dimensions need only be large enough to resolve the inter-plate distance and to contain an integer number of longitudinal vortex pairs or streaks. For a tilt $24^o$, the pattern wavelength and onset Reynolds number are in close agreement with experiment. The type of pattern depends on angle and isolated turbulent regions can be found, which may be similar to turbulent spots observed in Couette flows.

Computations were performed on the IBM SP4 computer of the IDRIS-CNRS supercomputer center, project 1119.

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