THE INSTABILITY OF A LOCALIZED VORTEX DISTURBANCE IN UNIFORM SHEAR FLOW

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Introduction

The similarity of the coherent structures (streaks and hairpin vortices) naturally occurring in different fully developed bounded turbulent shear flows as well as in transitional flows suggests the existence of a basic mechanism responsible for the formation of these structures, under various base flow conditions. The common elements for all such flows are the *shear of the base flow* and the presence of a localized vortical disturbance. The objective of the present numerical (and theoretical) study is to examine the capability of a *simple model of interaction, between a localized vortical disturbance and laminar uniform unbounded shear flow*, to reproduce the generation mechanism and characteristics of the coherent structures that naturally occur in turbulent bounded shear flows. This objective is shown schematically in Fig. 1.

Problem statement and methodology

The base flow is characterized by its shear $\boldsymbol{\Omega} = (0, 0, \Omega_z)$ and the localized disturbance by its vorticity field $\boldsymbol{\omega}$. Two geometries of the initial disturbance are considered: (i) a 'spherical' vortex ring (Gaussian vortex), the vorticity field of which is given by $\boldsymbol{\omega} = -\boldsymbol{p} \times \nabla \boldsymbol{F}$, $\boldsymbol{F} = (\pi^{1/2}\delta)^{-3} exp(-r_s^2/\delta^2)$, where $\boldsymbol{p}$ is a vector defining the disturbance magnitude and orientation, δ is a single characteristic disturbance length scale and r_s is a spherical radial coordinate; (ii) a horizontal (xz-plane) torus with a vorticity field: $\boldsymbol{\omega} = \{z, 0, -x\}^T A\, exp(-y^2/\delta^2) exp(-(r - r_o)^2/\delta^2)$, where x, y and z are the streamwise, normal and spanwise directions, respectively, A is a constant defining the strength of the disturbance, $r = \sqrt{x^2 + z^2}$ is a radial cylindrical coordinate and r_o and δ are two characteristic length scales associated with the radius and the thickness of the torus, respectively. The existence of two characteristic length scales allows us to vary the localized properties of the toroidal disturbance by changing the ratio r_o/δ, and to seek 'optimal' aspect ratio, for which the disturbance growth is maximal. (Toroidal disturbance may be considered as a limit of a Gaussian vortex for $r_o = 0$.)

For a given shape of the horizontal initial disturbance, two parameters govern the disturbance evolution: (i) the strength of the initial disturbance (ε) defined by the ratio between the maximum vorticity of the disturbance (ω_{max}) and the shear of the base flow (Ω), i.e. $\varepsilon = \omega_{max}/\Omega$ (for $\varepsilon \ll 1$, the disturbance evolution is governed by the linearized equations); (ii) the Reynolds number based on the characteristic length scale δ, i.e., $Re = \Omega\delta^2/\nu$.

To follow the evolution of the vortical structure, its shape is identified by the iso-surfaces of the vorticity magnitude as well as by the iso-surfaces of the positive second invariant of the velocity gradient tensor, capturing the regions of concentrated vorticity and swirling motion, respectively. The vortical structure is then characterized by its strength, W, measured by integrating the enstrophy over the entire volume, (i.e. $W = \int_V \parallel \boldsymbol{\omega} \parallel^2 dV$); its center, $\bar{X}_i$, defined as the center of its enstrophy (i.e. $\bar{X}_i = \int_V \parallel \boldsymbol{\omega} \parallel^2 x_i dV / W$) and by its inclination angle, α, (Fig. 2e) relative to the base flow.

Full unsteady incompressible Navier-Stokes equations were solved using the commercial CFD code 'FLUENT'.

Main results and conclusions

A temporal evolution of a toroidal vortex (with $r_o/\delta = 2$) is shown in Fig. 2. Similar results were obtained for other toroidal and Gaussian vortex disturbances. Accordingly, a small amplitude initial disturbance (linear case) eventually evolves into a streaky structure, whereas a large amplitude disturbance evolves into a hairpin vortex independent of its initial geometry. The main non-linear effects are: (i) the movement of the vortical structure relative to the base flow due to its self-induced motion (the centers of the vortical structure are indicated by the black bullets in Figs. 2 (d) and (e); (ii) the destruction of the vortex streamwise symmetry. In addition, for the small amplitude case, the induced velocity is mainly in a direction opposite to that of the base flow (Fig. 2 d), and therefore the vorticity vector strongly deviates from the direction of the vortical structure. Thus, the vortical structure cannot be represented as a vortex filament. However, as the amplitude of the disturbance is increased the vorticity lines follow more closely the vortical structure (Fig. 2e and other results not shown here).

The qualitative evolution of the disturbance is almost independent of its initial geometry, whereas the quantitative characteristics (i.e. vortex strength, center and inclination angle), strongly depend on the disturbance geometry. The Reynolds number has a negligible effect on the kinematics of the vortical structure, but does have a significant effect on its transient growth. The strength of the vortical structure is governed by the transient growth mechanism, which strongly depends on the disturbance geometry. For the toroidal disturbance there is an 'optimal' range of r_o/δ where the transient growth is maximal (Fig. 3). For this range the spanwise separation ($\approx 2r_o$) of two elongated vortical regions, expressed in terms of wall units, lies within the interval of $63 < y^+ < 88$, which corresponds well to the spanwise spacing of low-speed streaks (as well as the distance between hairpin vortex legs) in turbulent boundary layers. Finally, the above numerical results will be discussed with respect to recent theories concerning the evolution of localized disturbances.

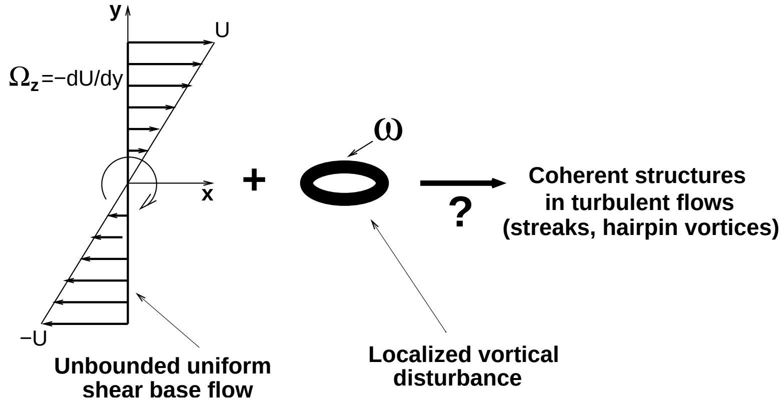

Figure 1. Schematic drawing of the research objective.

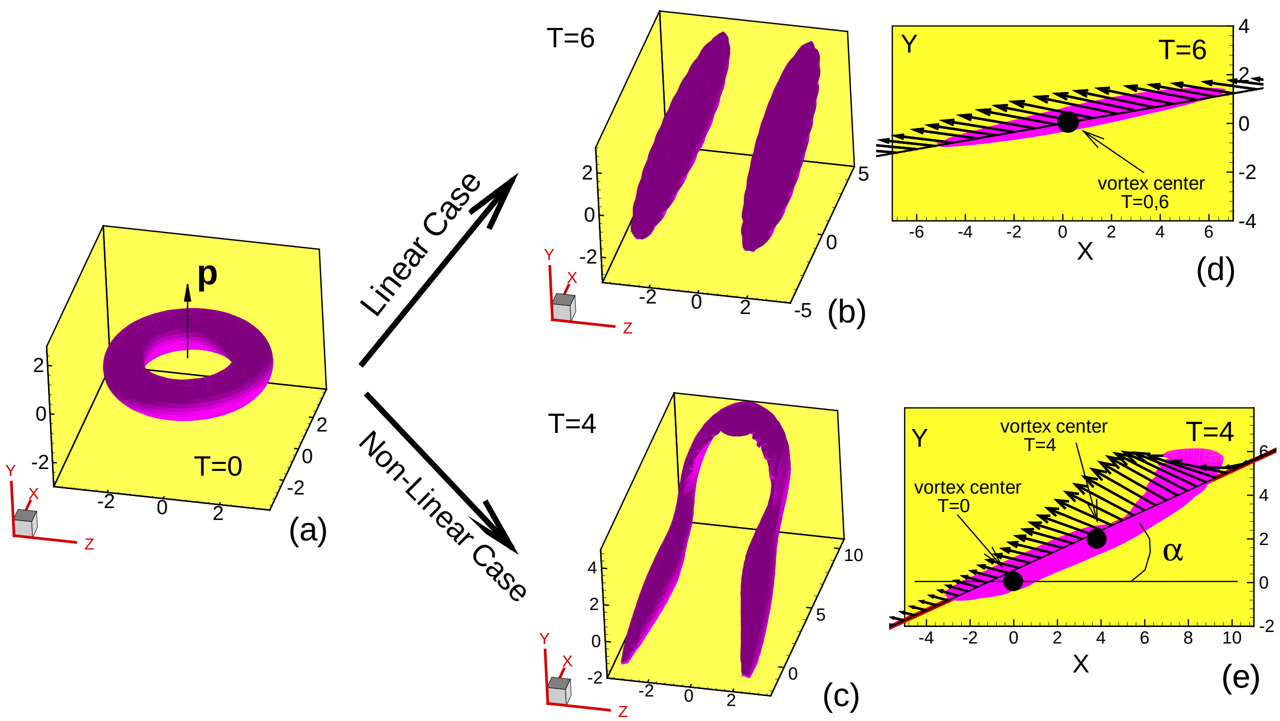

Figure 2. The evolution of a toroidal disturbance with $r_o/\delta = 2$ and $Re = 40$, shown by the iso-surfaces of the vorticity magnitude for $\| \boldsymbol{\omega} \| / \omega_{max} = 0.6$. (a) The initial disturbance, (b) streaky structure evolved from the toroidal disturbance with $\varepsilon = 0.375$, (c) hairpin vortex evolved from the toroidal disturbance with $\varepsilon = 4.5$, (d) & (e) projections on the xy plane of the iso-surfaces shown in (b) and (c), respectively, together with the velocity vectors along the axis of the vortical structure at the plane $Z = 0$ $(T = \Omega\, t;\;\; X = x/\delta,\, Y = y/\delta)$.

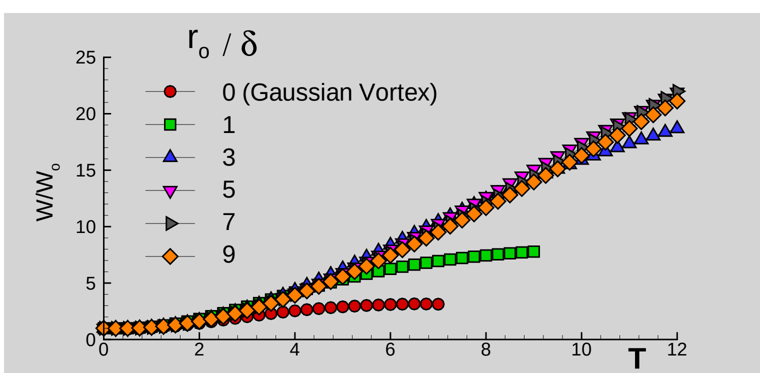

Figure 3. Temporal evolution of the enstrophy integral normalized by its initial value (W/W_o), $\;\varepsilon = 0.375,\;\; Re = 40.$