NUMERICAL INVESTIGATION OF CONVECTIVE REGIMES IN A PLANAR FILTRATIONAL CONVECTION PROBLEM

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Summary One-parameter families of steady-state convection regimes developing in the Darcy plane convection problem in a rectangular vessel are investigated numerically with increase of filtrational Rayleigh number and various aspect ratios of the container. The reason of the existence of these families is the cosymmetry. We consider Galerkin systems of various dimensions (up to 1000) for PDE approximation. The qualitative repetition of the bifurcations and consistency of the bifurcation parameter values was established by investigation of Galerkin's models of increasing dimensions for each set of the physical parameters. The loss of stability on a primary family, bifurcations of equilibrium families, periodic and chaotic regimes are studied. The fluid motion and heat transfer by convective regimes are also investigated.

INTRODUCTION AND MATHEMATICAL FORMULATION OF THE PROBLEM

In investigating convective motions of the fluid in a horizontal cylinder with an arbitrary shape of cross-section and with high heat-conducting side walls it had been found [1, 2] that a one-parameter family of equilibria was developed as a result of the first bifurcation. As shown in [2] the cause of the existence of the family was a nontrivial cosymmetry, the theory of which was developed last years. An analytical investigation of the one-parameter family of equilibria in the planar convection problem makes it possible to obtain asymptotics of the family in the neighborhood of its onset. The question about the evolution of the family at increasing the Rayleigh number is out of analytical treatment and requires numerical investigation as well as the analysis of occurrence and development of auto-oscillations.

We consider a rectangular container $D = [0, a] \times [0, b]$ filled with a porous medium and saturated by an incompressible fluid. The dimensionless equations of gravity convection of the fluid in the container uniformly heated from below have the form [1, 2]:

$$\left\{ \begin{array}{l} \Delta\psi = \theta_x, \\ \theta_t + \psi_y\theta_x - \psi_x\theta_y = \Delta\theta + \lambda\psi_x. \end{array} \right. \tag{1}$$

Here $\psi(x, y, t)$ is the stream function, $\theta(x, y, t)$ is the temperature deviation from the equilibrium profile. The parameter λ is the filtrational Rayleigh number. On the boundary of the container the Dirichlet boundary condition is specified

$$\theta\Big|_{\partial D} = \psi\Big|_{\partial D} = 0. \tag{2}$$

We can express ψ in terms of θ by solving the Dirichlet boundary value problem for the Poisson equation

$$\psi = G\theta_x, \tag{3}$$

where G is the corresponding Green's operator. As shown in [2, 3], problem (1), (2) has a cosymmetry determined by the right-hand side of equality (3).

The critical values of the spectral problem for the zero equilibrium are [3]

$$\lambda_{m,n} = 4\pi^2\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right), \qquad m, n = 1, 2, \ldots \tag{4}$$

All eigenvalues (4) are at least double and each transition of the bifurcation parameter λ through the value $\lambda_{m,n}$ corresponds to the bifurcation of the origin of a one-parameter family of equilibria. As shown in [2] the first critical value $\lambda_{1,1}$ is always double and a cycle of steady-state regimes branches off the quiescent state at $\lambda = \lambda_{1,1}$. All the equilibria of the family are neutrally stable along the cycle and asymptotically stable in the transversal directions, and their spectrum depends on the equilibrium coordinates and, therefore, this family cannot be an orbit of the action of any symmetry group [3].

In order to approximate problem (1)-(2) the Galerkin method is used. The numerical solution to the problem is found in the form

$$\psi = \sum_{i=1}^{nx}\sum_{j=1}^{ny} \psi_{i,j}(t)\phi_{i,j}(x, y), \quad \theta = \sum_{i=1}^{nx}\sum_{j=1}^{ny} \theta_{i,j}(t)\phi_{i,j}(x, y),$$
$$\phi_{i,j}(x, y) = \tfrac{2}{\sqrt{ab}} \sin\left(m_i \tfrac{\pi x}{a}\right) \sin\left(n_j \tfrac{\pi y}{b}\right). \tag{5}$$

Substitution (5) into (1) and corresponding projection operations lead to a system of ordinary differential equations of the order $N = nx \times ny$ for the functions $\theta_{i,j}$ [6, 7]. The special technique for calculation of one-parameter families of equilibria was developed in [7] wich based on the cosymmetric version of the implicit function theorem [2, 4] .

NUMERICAL RESULTS

Loss of stability on the family

The loss of stability on a one-parameter family of noncosymmetric equilibria of ordinary differential equations systems is extended on a parameter. It means, that in the beginning on a set there are equilibria with a neutral spectrum. Then these equilibria lose stability, and in their neighborhood there are arcs of unstable equilibria. With a modification of a bifurcation parameter the sizes of unstable arcs are increased, the amount of steady and unstable arcs can vary, both at the expense of origin new, and in an outcome of confluence already existing. We find that the loss of stability on a stable set of stationary regimes depends of vessel size and can be oscillatory or monotone and can happens simultaneously in two, four, six or eight points [5, 6, 10]. After the first loss of stability and further increasing of the parameter λ the set of unstable regimes grow by forming unstable arcs in the families. Further increase of λ may lead to appearence of unstable regimes in new regions of the family both with monotonic and oscillatory instabilities.

Stationary convective regimes

To each point of a one-parameter family of equilibria corresponds stationary driving of a liquid. Is shown, that the character of complication of steady stationary convective regimes depends on geometry of the container [6, 7, 10]. The evolution of convective regimes from the appearance of the family to its instability is the following: for a narrow container increasing of λ makes the structure of existing rolls more complicated, and new convective cells are formed inside the existing ones; for a wide vessel new convective rolls appear. Both local and average heat transfer of the regimes in a wide vessel do not differ much neither qualitatively nor quantitatively. This difference is essential for a narrow vessel since there exist regimes with high and low heat transfer at the top and at the bottom of the vessel.

Bifurcations of families

The series of bifurcations of one-parameter families of equilibria was found out: birth of a new family from already existing, intersection and join of equilibrium curve, origin of a cycle of equilibria 'from an air' etc [6, 7, 10]. The bifurcations of families are connected with different equilibrium families both with primary stable and with unstable. All enumerated bifurcations take place in Galerkin systems of all considered dimensions. We found many variants of bifurcations studied theoretically in [8].

Periodic and chaotic regimes

The periodic and chaotic regimes was founded for different values of λ and sizes of D. The steady periodic regimes can occurs by two ways: as a result of special cosymmetric Hopf bifurcation or as a result of Euler bifurcation that depends on container size. For some values of parameter transition from stationary movement of a liquid at once to chaotic movement is possible. Both periodic and chaotic modes can coexist with steady stationary regimes.

Selection of stable regimes

It is natural to ask which stable stationary regimes of one-parameter family will be realized in a experiment. In the computer experiment, the initial points were distributed uniformly on a small sphere centered on the zero equilibrium for values of λ. We found that two regimes can be realized from the initial conditions near mechanical equilibrium. The selection mechanism was pretty obvious in this case and connected with phase space structure.

References

[1] Lyubimov D.V.: Convective motions in a porous medium heated from below, *Zh. Prikl. Mekh. Tekh. Fiz.* **2**:131-137, 1975.

[2] Yudovich V.I.: Cosymmetry, degeneration of solutions of operator equations, and onset of filtration convection, *Mat. Zametki*, **5, Vol.49**:142-148, 1991.

[3] Yudovich V.I.: Secondary cycle of equilibria in a system with cosymmetry, its creation by bifurcation and impossibility of symmetric treatment of it, *Chaos*, **2, Vol.5**:402-411, 1995.

[4] Yudovich V.I.: Implicit function theorem for cosymmetric equations, *Mat. Zametki*, **2, Vol. 60.**:313-317, 1996.

[5] Govorukhin V.N.: Numerical Simulation of the Loss of Stability for Secondary Steady Regimes in the Darcy Plane-Convection Problem, *Doklady Physics*, **12, Vol.43**:806-808, 1998.

[6] Govorukhin V.N.: Analysis of families of secondary steady-state regimes in the problem of plane flow through a porous medium in a rectangular vessel, *Fluid Dynamics*, **5, Vol.34**:652-659, 1999.

[7] Govorukhin V.N.: Calculation of one-parameter families of stationary regimes in a cosymmetric case and analysis of plane filtrational convection problem, *Notes on Numerical Fluid Mechanics*, **Vol. 74**:133-144, 2000.

[8] Kurakin L.G. and Yudovich V.I.: Bifurcations accompanying monotonic instability of an equilibrium of a cosymmetric dynamical system. *Chaos* **V. 10, N. 2**:311–330, 2000.

[9] Govorukhin V.N. and Schevtschenko I.V.: Numerical solution of Planar Darcy convection problem on a distributed-memory computer, *Computational Technologies*, **1, Vol 6.**:3-12, 2000.

[10] Govorukhin V.N. and Schevtschenko I.V.: Numerical investigation of second bifurcation transitions in a plane filtrational convection problem, *Mech, Zhidkosty i Gaza*, **N5**:115-128, 2003.