A SINGULAR VALUE ANALYSIS OF BOUNDARY LAYER CONTROL

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Summary Several approaches for boundary-layer control are analyzed from a linear system point of view. The singular value decomposition (SVD) is applied to the linearized Navier-Stokes system in the presence of control. The performance of control is examined in terms of the largest singular values, which represent the maximum disturbance energy growth ratio attainable in the linear system under control. The maximum growth ratio is shown to be less in controlled systems than in the uncontrolled system only when control parameters are within a certain range of values. The SVD analysis of various flow controls shows a similarity between the trend observed in the SVD analysis (linear) and that observed in direct numerical simulations (nonlinear), thus reaffirming the importance of linear mechanisms in the near-wall dynamics of turbulent boundary layers. The present study illustrates that the SVD analysis can be used as a guideline for designing controllers for drag reduction in turbulent boundary layers.

INTRODUCTION

There have been increased activities in developing efficient and robust controllers using modern control theory for viscous drag reduction in turbulent boundary layers. Although turbulent flows are in general governed by nonlinear dynamics, some of these new approaches specifically target a linear mechanism that has been identified to be responsible for high-skin friction in turbulent boundary layers. The fact that a linear mechanism plays an important role in this nonlinear turbulent flow allows us to investigate the flow from a linear system theory point of view. Indeed, recently several investigators have reported successful applications of linear controllers derived from linear control theory[1][2], but many fundamental and challenging questions have been raised in the course of applying linear optimal control to turbulent boundary layers.

MATHEMATICAL FORMULATIONS

The traditional linear analysis, which predicts whether a linear system is stable or unstable based on the eigenvalues of the system, is inadequate in explaining transient—nonetheless quite substantial—growth of the kinetic energy of certain disturbances in otherwise a stable system. This transient growth is due to non-normality of the linearized Navier-Stokes system. Eigenfunctions of a non-normal system are not orthogonal to each other, and as such the kinetic energy of certain disturbances can grow before its ultimate decay even in a linear system with no unstable eigenfunctions. Since the transient growth is due to a linear mechanism, its behavior can be analyzed through a singular value decomposition (SVD) of the system operator [3], with which the amplification factor of the so-called optimal disturbance could be determined. We hypothesize that the SVD analysis is also appropriate for examining performance of controllers for turbulent boundary layers. Effective controllers must reduce the non-normality of the flow system, since it is also believed to be responsible for sustaining near-wall turbulence structures, which in turn are responsible for high skin-friction drag in turbulent boundary layers.

State-space representation

The first step to apply the SVD analysis is to formulate the governing equations in terms of the state-space representation as is done for other linear optimal controller design. The linearized Navier-Stokes equations with control input can be written in the following state-space representation:

$$\frac{d\mathbf{x}}{dt} = A\mathbf{x} + B\mathbf{u}, \tag{1}$$

$$\mathbf{u} = -K\mathbf{x}\,, \tag{2}$$

where $\mathbf{x}$ and $\mathbf{u}$ represent the state-space and control input vectors (wall-transpiration in this study), respectively, and A, B and K represent the system, actuation and control-gain matrices, respectively. The control-gain matrix K for an LQR controller is determined by solving an algebraic Riccati equation[1][2], while that for the opposition control suggested by Choi *et al.*[4] is easy to construct (especially when $\mathbf{x}$ represents a collocation vector) once the detection-plane location is known.

Singular Value Decomposition

To analyze the transient energy growth, we consider the ratio of the kinetic energy of a disturbance at a given time (τ) to the disturbance energy at $t = 0$. After a simple mathematical procedure used by Reddy and Henningson[3], we can define the growth ratio function, $G(\tau)$, as

$$G(\tau) = \sup_{\mathbf{x}(\cdot,0)\neq 0} \frac{||\mathbf{x}(\tau)||^2}{||\mathbf{x}(0)||^2} = \sup_{\mathbf{x}(\cdot,0)\neq 0} ||\mathbf{F}\exp[(\mathbf{A}-\mathbf{BK})\tau\,]\mathbf{F}^{-1}||_2^2\,, \tag{3}$$

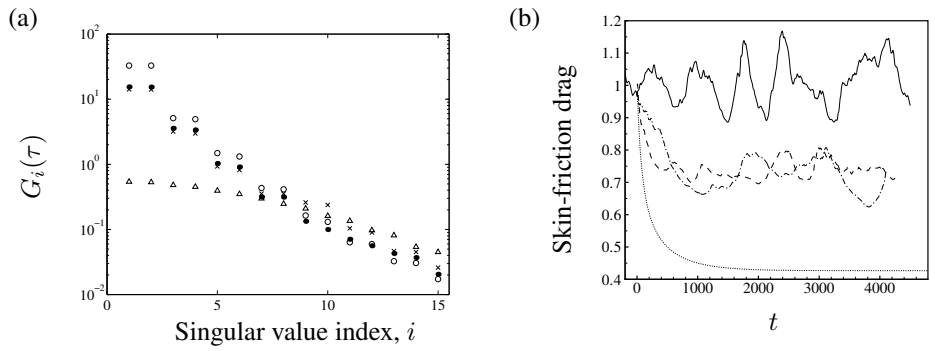

Figure 1. (a) Comparison of singular values for a wavenumber ($k_x = 0, k_z = 6$): ○ , no control; ●, opposition control; ×, LQR control; △, virtual flow. (b) Mean skin-friction drag history: ——— , no control; - - - - , opposition control, -·-·- , LQR control; ····· , virtual flow. Here, Re_τ=100.

where the quantity $||\mathbf{x}||^2$ represents the kinetic energy of $\mathbf{x}$, $||\cdot||_2$ is the 2-norm (Euclidian Norm), τ is the given time mentioned above, and the Hermitian matrix $\mathbf{F}$ is defined in terms of the square-root matrix of an inner product in discrete space with sufficient grid points[3]. The 2-norm of a matrix can be easily computed from the SVD of the matrix. The largest $G(\tau)$ value represents the maximum energy growth ratio at τ.

RESULTS

We have performed the SVD analysis of various linear-quadratic-regulator (LQR) controllers we have developed as well as the so-called opposition control used by Choi *et al* [4], in the hope that it could shed new light into these control methods. Note that these methods have been designed to achieve a similar goal, *i.e.*, interfering the interactions between the near-wall streamwise vortices and the wall in order to reduce skin-friction drag in turbulent boundary layers.

Figure 1 (a) shows distributions of singular values for the disturbance energy growth with an appropriate time scale. It can be seen that the singular values corresponding to the opposition control (with the detection-plane location at $y^+ = 15$) and an LQR-controller (minimizing the disturbance energy growth) are much smaller than those corresponding to the uncontrolled system. The singular values corresponding to the opposition-controlled system with different detection-plane locations were higher than those shown here, corroborating with the numerical observation that $y^+ = 15$ is the optimal location. Similarly, an optimal range of control parameters can also be found in LQR control cases [6]. Also, shown are the singular values for a virtual flow [5], in which the linear coupling term between the Orr-Sommerfeld and the Squire equations was removed. Note that all singular values for this case are less than one, indicating that no transient growth in the virtual flow. From these distributions of singular values, one would expect that the virtual flow would be most effective in reducing the skin-friction drag in turbulent boundary layers.

In order to examine the applicability of the above SVD analysis, which is based on the linearized Navier-Stokes system, to fully nonlinear turbulent flows, we applied these controllers to direct numerical simulations of a turbulent channel flow at the same Reynolds number. Figure 1(b) shows the time evolution of mean skin-friction drag in the channel with various controllers. Note that the case without the linear coupling term (virtual flow)results in complete laminarization, consistent with the SVD analysis. Other results are also consistent with the SVD analysis, demonstrating that the SVD analysis is a viable tool in predicting the performance of a controller in the nonlinear turbulent channel flow.

CONCLUSION

We have shown that the SVD analysis could be used to gain useful information on the performance of certain controllers. It could be used in optimizing control parameters without actually performing expensive nonlinear computations. Other issues, such as the effect of using the evolving mean flow as control applied to a nonlinear flow system (also known as gain scheduling) and high Reynolds number limitation, will be investigated through the SVD analysis.

References

[1] Joshi S. S., Speyer J. L., Kim J.: A systems theory approach to the feedback stabilization of infinitesimal and finite-amplitude disturbances in plane Poiseuille flow. *J. Fluid Mech.* **332**, 157–184, 1997.

[2] Lee K. H., Cortelezzi L., Kim J., Speyer J. L.: Application of reduced-order controller to turbulent flows for drag reduction. *Phys. Fluids* **13** (5), 1321–1330, 2001.

[3] Reddy S. C., Henningson D. S.: Energy growth in viscous channel flows. *J. Fluid Mech.* **252**, 209–238, 1993.

[4] Choi H., Moin P., Kim J.: Active turbulence control for drag reduction in wall-bounded flows.*J. Fluid Mech.* **262**, 75–110, 1994.

[5] Kim J., Lim J.: A linear process in wall-bounded turbulent shear flows. *Phys. Fluids* **12** (8), 1885–1888, 2000.

[6] Lim J., Kim J.: A singular value analysis of boundary layer control. *Phys. Fluids* submitted.