

SELF-PROPULSION OF AN OSCILLATORY WING

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Summary In this paper we show that the oscillatory motion of an airfoil (wing) in an incompressible inviscid fluid can determine the apparition of a propulsive force. To this aim we discretize the lifting surface equation and calculate the pressure coefficients. Integrating, we calculate the average drag coefficient which for a range of values of the frequency may happen to be negative. The oscillatory motion of a wing can model the motion of fishes and birds. It is also of great importance in aeroelasticity.

THE INTEGRAL EQUATION

We consider a system of coordinates $Ox^{(1)}y^{(1)}z^{(1)}$ related to the wing and we introduce the dimensionless space coordinates $(x, z) = \left(\frac{x^{(1)}}{a}, \frac{z^{(1)}}{a}\right)$, taking the wing length a as reference length along the vertical direction ($Oz^{(1)}$ -axis direction) and along the direction of the unperturbed uniform flow ($Ox^{(1)}$ -axis direction). We introduce also the dimensionless space coordinate $y = \frac{y^{(1)}}{b}$, taking the wing half-span b as reference length along the $Oy^{(1)}$ -axis direction. We denote by $D^{(1)}$ the wing projection on the $Ox^{(1)}y^{(1)}$ -plane. Let

$$0 = F(x^{(1)}, y^{(1)}, z^{(1)}) = z^{(1)} - h^{(1)}(x^{(1)}, y^{(1)}) \exp(i\omega t) ; (x^{(1)}, y^{(1)}) \in D^{(1)}, \quad (1)$$

$$\left| h^{(1)} \right| \ll 1, \quad \left| \frac{\partial h^{(1)}}{\partial x^{(1)}} \right| \ll 1,$$

be the equation of the oscillating wing (we neglect the thickness of the wing).

In the sequel $\rho_0 \text{Re}(f \exp(i\omega t))$ is the jump of the pressure over the oscillating wing (ρ_0 is the density of the fluid at rest), ω is the frequency of the oscillation, V_0 is the translation velocity of the unperturbed flow with respect to the $Ox^{(1)}y^{(1)}z^{(1)}$ frame of reference, $\varpi = \frac{b}{a}$ is the aspect ratio.

We introduce the dimensionless functions and variables

$$h(x, y) = \frac{h^{(1)}(x^{(1)}, y^{(1)})}{a}, \quad \tilde{\omega} = \frac{\omega a}{V_0}, \quad \tilde{f}(x, y) = \frac{f(ax, by)}{V_0^2}, \quad x_0 = x - \xi, \quad y_0 = y - \eta,$$

In the framework of the linearized theory the lifting surface equation for oscillatory airfoils is [1]:

$$\begin{aligned} \frac{\varpi}{4\pi} \int \int_D \tilde{f}(\xi, \eta) \exp(-i\tilde{\omega}x_0) \left(\int_{-\infty}^{x_0} \frac{\exp(i\tilde{\omega}u)}{(u^2 + \varpi^2 y_0^2)^{3/2}} du \right) d\xi d\eta = \\ = - \left(\frac{\partial h(x, y)}{\partial x} + i\tilde{\omega}h(x, y) \right), \quad D = \{(x, y); (ax, by) \in D^{(1)}\}. \end{aligned} \quad (2)$$

In the sequel we are going to put into evidence the singularities of the kernel. We have

$$\int_{-\infty}^{x_0} \frac{\exp(i\tilde{\omega}u)}{(u^2 + \varpi^2 y_0^2)^{3/2}} du = K^{(1)}(x, y; \xi, \eta) + K^{(2)}(x, y; \xi, \eta) + K^{(3)}(x, y; \xi, \eta),$$

where the kernel

$$K^{(1)}(x, y; \xi, \eta) = \frac{1}{\varpi^2 y_0^2} \left(1 + \frac{x_0}{\sqrt{x_0^2 + \varpi^2 y_0^2}} \right),$$

has a strong singularity, the kernel

$$K^{(2)}(x, y; \xi, \eta) = -\frac{i\tilde{\omega}}{\sqrt{x_0^2 + \varpi^2 y_0^2}} + \frac{\tilde{\omega}^2}{2} \ln \left(-x_0 + \sqrt{x_0^2 + \varpi^2 y_0^2} \right),$$

has weak singularities and the kernel

$$\begin{aligned}
K^{(3)}(x, y; \xi, \eta) = & \frac{\tilde{\omega}}{\varpi |y_0|} \left(K_1(\tilde{\omega} \varpi |y_0|) - \frac{1}{\varpi^2 y_0^2} - \frac{\tilde{\omega}^2}{2} \ln \frac{\tilde{\omega} \varpi |y_0|}{2} \right) + \\
& + i \frac{\pi}{2} \frac{\tilde{\omega}}{\varpi |y_0|} \left(I_1(\tilde{\omega} \varpi |y_0|) - L_{-1}(\tilde{\omega} \varpi |y_0|) + \frac{2}{\pi} \right) + \frac{\tilde{\omega}^2 x_0}{2 \sqrt{x_0^2 + \varpi^2 y_0^2}} + \frac{\tilde{\omega}^2}{2} \ln \frac{\varpi}{2} + \\
& + \int_0^{x_0} \frac{\exp(i \tilde{\omega} u) - 1 - i \tilde{\omega} u + \frac{1}{2} \tilde{\omega}^2 u^2}{(u^2 + \varpi^2 y_0^2)^{3/2}} du,
\end{aligned}$$

has no singularity. (I_1 and K_1 are Bessel functions and L_{-1} is a Strouve function.)

THE PROPULSIVE FORCE FOR A DELTA WING

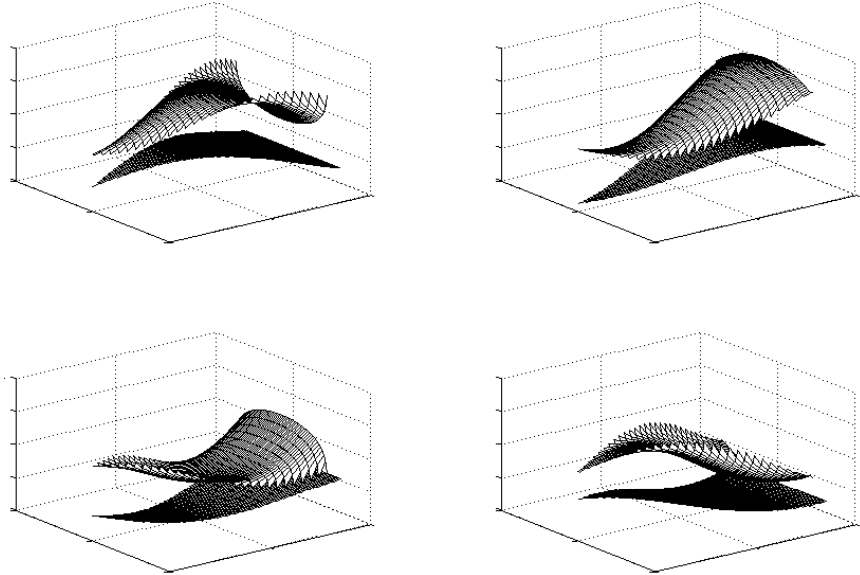
Employing for each type of kernel adequate quadrature formulas, we discretize (2) obtaining the values of $\tilde{f}$ in the nodes of the grid. With the the formulas

$$C_p(x^{(1)}, y^{(1)}, t) = \text{Re}[\tilde{f}(x, y) \exp(i \omega t)], \quad C_D(t) = -2 \int \int_D n_x C_p(ax, by, t) dx dy, \quad \tilde{C}_D = \frac{1}{T} \int_0^T C_D(t) dt$$

where T is the period of the oscillation, we calculate the pressure coefficient field, the drag coefficient and the average drag coefficient. Considering the oscillating delta wing whose equation is

$$h(x, y) = \alpha \exp(i \tilde{\omega}_1 x), \quad (x, y) \in D = \{(x, y); 0 < y < |x|, 0 < x < 1\},$$

we calculate the average drag coefficient and we notice that if $\tilde{\omega}$ surpasses a certain critical value, the average drag coefficient becomes negative i.e. there appears a *propulsive force*. In the following figure we present the oscillating wing (below) and the pressure coefficient fields (above) for the values of the nondimensional time $\frac{V_0}{a} t \in \{1, 1, 2, 4\}$ and for the nondimensional frequencies $\tilde{\omega} = \pi/2$, $\tilde{\omega}_1 = 6\pi/5$. We considered $\varpi = 1/8$.



References

- [1] D. Homentcovschi, Theory of the lifting surface in unsteady motion in an inviscid fluid, *Acta Mechanica* **27**, 205-216, 1977.