INITIAL ENERGY DISSIPATION MECHANISM AT CRACK TIP AND THE DUCTILE TO BRITTLE TRANSITION

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Summary Criteria which determine the initial energy dissipation mechanism that is activated at or near a crack tip are derived. The possible mechanisms considered are cleavage, crack tip dislocation nucleation and Frank-Read source activation near the tip. The criteria can be succinctly expressed in graphical form. Emphasis is on quantitatively comparing the criteria to experiments in the literature.

INTRODUCTION

The brittle to ductile transition of a crystalline material that contains a crack is typically defined as an abrupt change in the overall capacity of the material to dissipate energy by means of irreversible deformation processes. It is thought that the transition is triggered by a change, at or near the crack tip, from one type of irreversible deformation mechanism to another. Traditionally two distinct competitions between deformation mechanisms have been considered in an attempt to explain the brittle to ductile transition. One approach invokes the competition between dislocation nucleation from a crack tip and cleavage failure. The second approach assumes that the mobility of dislocations plays the key role, which sets up a competition between cleavage failure and plastic deformation due to pre-existing dislocations in the material near a crack tip.

A recent analysis [1] has reconciled the two approaches by considering the energetic competitions among the following three energy dissipation mechanisms: cleavage, dislocation nucleation from a crack tip, and dislocation nucleation from a Frank-Read source near a crack tip (which implicitly requires dislocation mobility). The result is a set of three criteria involving dimensionless parameters which determine whether cleavage, crack tip dislocation nucleation or Frank-Read dislocation nucleation is the initial energy dissipation mechanism that is activated at or near a crack tip upon the commencement loading. The dimensionless parameters that enter into the criteria are $b\rho_{disl}^{1/2}$, τ_p/μ, $\gamma_s/\mu b$, and $\gamma_{us}/\mu b$, where b is the Burgers vector, ρ_{disl} is dislocation density, τ_p is the Peierls stress which here is defined as the minimum resolved shear stress necessary to move a dislocation through a crystal lattice at the temperature of interest, γ_s is surface free energy, γ_{us} is unstable stacking energy, and μ is elastic shear modulus.

Since three energy dissipation mechanisms are considered, the brittle-ductile behaviour of a material can be grouped into three classifications. If cleavage is the initial energy dissipation mechanism, the material is said to be intrinsically brittle. If crack tip dislocation nucleation is the initial energy dissipation mechanism the material is intrinsically ductile. If dislocation nucleation from a Frank-Read source is predicted to occur at a lower load level than either cleavage or crack tip dislocation, the material is said to possess extrinsic ductility. Herein, the initial energy dissipation criteria from [1] are generalized and it is demonstrated that the criteria have a straightforward graphical interpretation. It is shown that the resulting predictions compare favourably to experiments reported in the literature.

THEORY

The applied crack tip energy release rate to activate Frank-Read (FR) sources nearest the tip is derived. The derivation is elementary and necessarily approximate, but suffices to obtain at least an order of magnitude estimate of the critical applied load. It is well-known that the resolved shear stress necessary to activate a FR dislocation source, τ_{FR}, scales as $\tau_{FR} = \alpha\mu b/L$, where L is considered to be the mean distance between impediments to motion of the pre-existing dislocation and α is a dimensionless constant with magnitude $1/4 < \alpha < 10$, depending upon type of material. In pure single crystals, the distance between impediments scales as $L \approx \rho_{disl}^{-1/2}$, which corresponds to the mean distance between dislocations. Thus, $\tau_{FR} = \alpha\mu b\rho_{disl}^{1/2}$, which is dimensionally consistent with the overall macroscopic yield stress of face-centered cubic single crystals. This expression accounts only for the contribution of elastic energy due to increase in dislocation line length as the dislocation bows past the impediments, which is appropriate for materials with a very small Peierls stress, τ_p. However for classes of materials for which the Peierls stress is not sufficiently small, the effect of the Peierls stress must be taken into account. Under such conditions, it is appropriate to express the critical resolved shear stress for FR source activation as the sum of the contributions from both the increase in dislocation line length as well as the Peierls stress, which yields

$$\tau_{FR} = \frac{\alpha\mu b}{L} + \tau_p . \tag{1}$$

It is not possible to define *a priori* the precise position of FR sources near the crack tip. Nevertheless, it is well-accepted that the distance between dislocations, and hence between FR dislocation sources, scales as $\rho_{disl}^{-1/2}$. Therefore we assume that a FR dislocation source exists at a radial distance $r \approx \rho_{disl}^{-1/2}$ from a crack tip. The source consists of a pre-existing edge dislocation that spans two impediments and is parallel to the crack front. The spacing between the two impediments also scales as $L \approx \rho_{disl}^{-1/2}$. We assume the material to be fully annealed so that the Burgers vectors of all other surrounding pre-existing dislocations do not have any preferred orientation. Thus the net stress due to all other surrounding dislocations at any particular position is, on average, zero. Likewise the net image stress on a dislocation at any particular position due to all surrounding dislocations is, on average, zero. Therefore the dislocation of the FR source interacts only with the stress field of the crack tip and with its own image force. The interactions can be quantified in terms of the Peach-Koehler (PK) force, f, defined as $f = \tau b$, where τ is the resolved shear stress on the dislocation slip plane in the direction parallel to the Burgers vector. The FR source is activated once the sum of the PK forces from the crack tip and image force equals the force necessary to overcome $\tau_{FR} b$ from Eq.(1). Finally for simplicity we assume that the dislocation exists on a slip plane which intersects the crack tip (this assumption is not necessary, and does not affect the order of magnitude of the result). Rice and Thomson [2] discussed the PK forces on a dislocation due to a crack tip and its image force under the same conditions assumed here. Setting the sum of the PK force from the crack tip and from the image force equal to $\tau_{FR} b$ yields

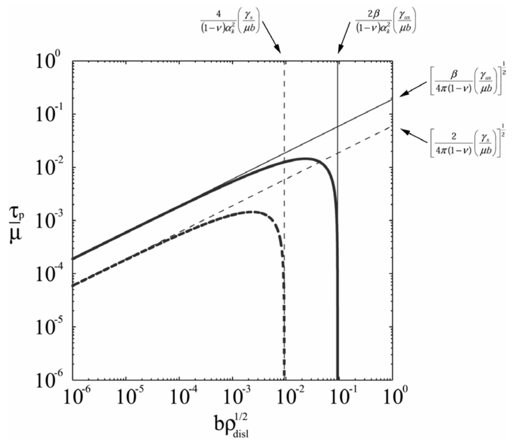
$$\frac{K_I b}{\sqrt{8\pi r}} - \frac{\mu b^2}{4\pi(1-\nu)r} = \frac{\alpha\mu b^2}{L} + \tau_p b \tag{2}$$

where K_I is the Mode-I stress intensity factor, ν is Poisson's ratio and r is the radial position of the dislocation relative to the crack tip. The first term on the left side of Eq.(2) is from the singular stress field of the crack tip and the second term represents the image force. It should be noted that the trigonometric functions normally associated with the crack tip term are approximated as unity, which does not affect the order of magnitude of the result. Substituting $r \approx \rho_{disl}^{-1/2}$ and $L \approx \rho_{disl}^{-1/2}$ and solving for K_I and expressing in terms of the applied energy release rate, G, yields

$$G_{FR} = \frac{(1-\nu)\alpha_k^2}{2}\mu b^2 \rho_{disl}^{1/2}\left[1 + \frac{\sqrt{8\pi}}{\alpha_k}\left(\frac{\tau_p}{\mu}\right)\left(\frac{1}{b\rho_{disl}^{1/2}}\right)\right]^2 \tag{3}$$

where $\alpha_k = \sqrt{8\pi}\{\alpha + 1/[4\pi(1-\nu)]\}$. This is an order of magnitude estimate for the energy release rate at which a FR source near a crack tip is expected to be activated.

It is well-known that the energy release rate for cleavage is $G_{CL} = 2\gamma_s$ and the energy release rate for crack tip dislocation emission [2] is $G_{CT} = \beta\gamma_{us}$, where β is a dimensionless constant. Thus it is evident that FR source activation will be the initial energy dissipation mechanism if both $G_{FR}/G_{CT} < 1$ and $G_{FR}/G_{CL} < 1$ hold. These criteria are plotted to obtain the figure to the right, where the two criteria are plotted as heavy lines. If the values of $b\rho_{disl}^{1/2}$ and τ_p/μ lie to the lower left of both heavy lines, FR sources are predicted to be activated prior to either cleavage or crack tip dislocation nucleation. The thin lines indicate the values of the asymptotes which serve to establish approximate ranges of $b\rho_{disl}^{1/2}$ within which a material is predicted to be extrinsically ductile as in [1].



A recent paper by Chan [4] assembled experimentally obtained fracture toughness data of Nb-Ti-Cr-Al solid solution alloys as a function of the Peierls energy barrier, which is related to τ_p. In terms of the quantities employed herein, Chan posited the experimentally-determined criterion $\gamma_s/\tau_p b > 20$ for dislocation activity near a crack tip to cause a substantial increase in fracture toughness. The criterion $G_{FR}/G_{CL} < 1$ can be rearranged in the limit of $\tau_p/\mu >> b\rho_{disl}^{1/2}$ to obtain a necessary criterion for extrinsic ductility to be $\gamma_s/\tau_p b > \sqrt{2\pi\,((1-\nu)/b\rho_{disl}^{1/2})(\gamma_s/\mu b)}$. Using appropriate values of the variables listed in [4], one obtains the criterion $\gamma_s/\tau_p b > 13$ when $\rho_{disl} = 10^{12}\,m^{-2}$, which is a reasonable assumed initial dislocation density for the experiments.

CONCLUSIONS

Criteria to determine the initial energy dissipation mechanism at or near a crack tip are derived which have a straightforward graphical interpretation. It is shown that the criteria quantitatively compare quite favourably to experiment.

References

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