NUMERICAL APPROACH FOR DYNAMIC FRACTURE IN PIEZOELECTRIC SOLIDS

José Domínguez*, Andrés Sáez*, Felipe García-Sánchez **

*Continuum Mechanics Department, Univesidad de Sevilla, Camino de los Descubrimientos s/n, 41092-Sevilla, Spain

**Civil Engineering Department, Universidad de Málaga, Plaza de El Ejido s/n, 29013-Málaga, Spain

Summary A mixed boundary element approach for two-dimensional dynamic piezoelectric fracture mechanics problems is presented. The numerical approach is based on displacement and traction integral equations for external and crack boundaries, respectively. Integral equations for time harmonic problems are considered. Transient problems are analyzed by means of the FFT. Curved and quarter-point elements are used. Stress and electric displacement intensity factors are evaluated from nodal values next to the crack tip. The present BE approach has allowed for solution of a variety of crack problems including curved cracks. Some of these problems had never been studied before.

PROBLEM STATEMENT

Piezoelectric materials are brittle and due to manufacturing process and complex electromechanical loads they are likely to develop cracks. The understanding and evaluation of the fracture process in piezoelectric materials is crucial to the advancement of modern intelligent material systems. Among the most significant publications on the field of piezoelectric materials fracture mechanics in terms of integral equations is the Ph. D. Thesis of Deeg [1].

In this paper, a general mixed BE formulation based on displacement and traction integral equations for 2-D cracked piezoelectric media under static and time harmonic load is presented. A new regularization procedure based on a simple change of variables is presented. All the strongly singular or hypersingular integrals are transformed into regular integrals and simple singular integrals with known analytical solution. The traction and normal electric displacement integral representations are written for the crack surface and the displacement and electric potential integral representation for the external boundaries. The basic variables are opening displacement and electric potential discontinuity along the crack, and displacement, electric potential, traction and normal electric displacement, on external boundaries. Quadratic quarter-point elements are used to represent the crack opening displacement (COD) and electric potential discontinuity near the crack tip. Standard curved or straight quadratic elements are used for the rest of the crack and external boundaries. Stress and electric displacement Intensity Factors (ESIF) are computed in a direct way from the COD and the electric potential discontinuity at a point extremely close to the tip.

NUMERICAL RESULTS

To show the use of the current procedure for curved crack geometries, the problem of a crack with circular arch shape in an unbounded domain is studied. The crack region is under a uniform static far field stress or electric displacement (Figure 1) along the material and crack axis of symmetry. The material is a PZT-4 ceramic. Cracks with different semi-angles θ are analysed. The BE mesh consists of eight curved quadratic elements and two very small (arch length/30) quarter-point straight elements at the tips (Figure 1). Computed values of K_I, K_{II} and K_{IV} when the load is a uniform electric displacement are shown in Figure 2, where the ratio between piezoelectric and dielectric parameters χ is used for normalization purposes. These results show the great influence that crack curvature has on ESIF. To the authors' knowledge, no solution for the curved crack problem in piezoelectric solids has been published up to the present.

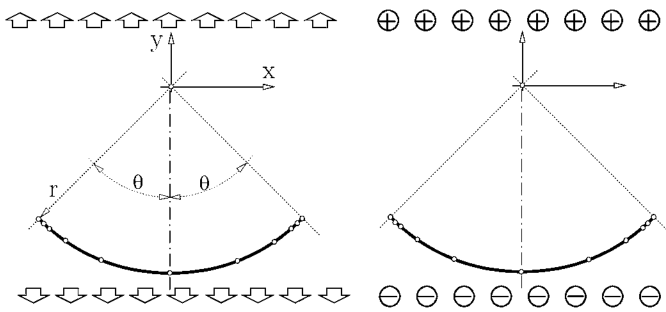

Figure 1. Geometry, mesh and loading conditions for a circular arch crack

A straight crack in an infinite piezoelectric plane under time harmonic waves impinging along the material axis of symmetry is studied next. Figure 3 shows the mode-I SIF versus dimensionless frequency. Results are in good agreement with those obtained by Shindo [2] for the same problem using a semi-analytical procedure. Time variation of K_I for a step load is shown in Figure 4, where time has been normalized as in Ref. [3]. Note that the peak in K_I is similar to that shown by the exact solution for elastic isotropic solids. Other numerical examples will be presented during the conference.

CONCLUSIONS

A general mixed boundary element formulation for static and dynamic crack problems in 2-D piezoelectric solids has been presented. The obtained results show the robustness, generality and simplicity of the procedure. The present results are in good agreement with solutions presented by other authors in the cases where those solutions exist.

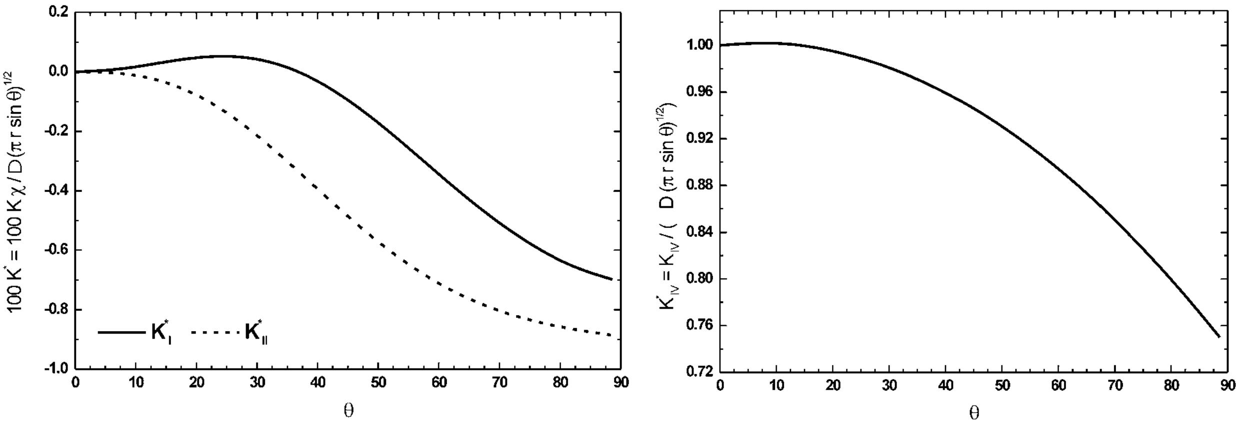

Figure 2. (i) Mode I and mode II SIF and (ii) Electric displacement intensity factor for circular crack in piezoelectric solid under uniform far field electric displacement

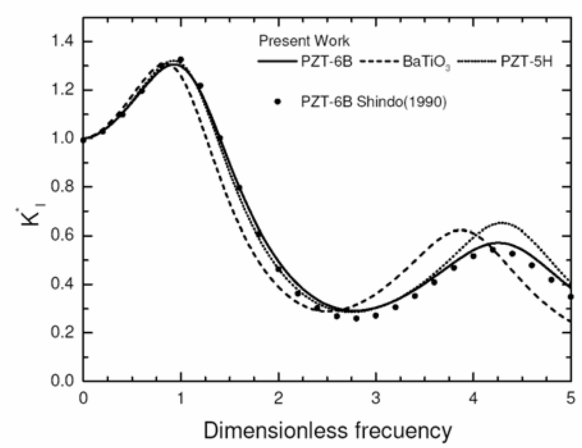

Figure 3. Mode I SIF versus dimensionless frequency for a Griffith crack under a plane harmonic wave

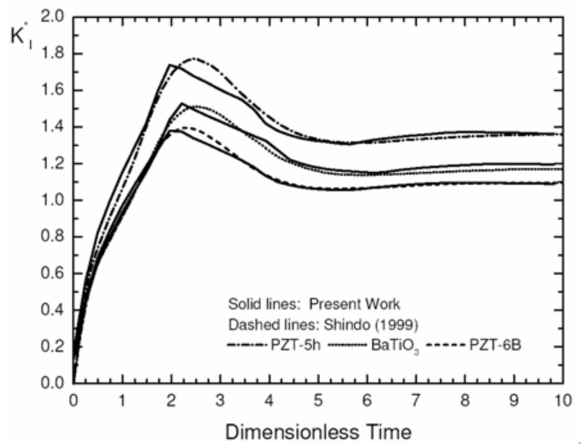

Figure 4. Mode I SIF versus dimensionless time for a Griffith crack under the action of normal impact loading

References

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[3] Shindo Y., Narita F., Ozawa E.: Impact response of a finite crack in an orthotropic piezoelectric ceramic. *Acta Mechanica* **137**: 99-107, 1999.