

CRACK ARREST MODEL FOR CRACKED PIEZOELECTRIC PLATE –A MODIFIED DUGDALE APPROACH

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Summary A crack arrest model based on Dugdale model is proposed for a poled piezoelectric plate weakened by a straight hairline crack. Two cases are investigated when crack lies perpendicular and along the poling direction. In each case the mechanical plastic zones develop, due to remotely applied tension and electric field, are being closed by linearly varying yield point stress. In each case problem is solved using Dugdale hypothesis and complex variable technique.

Research on piezoelectric materials has gathered impetus in last two decades because of their utility as sensors and actuators in smart structure systems. Piezoelectric materials obey Hooke's law and exhibit brittle fracture behavior, consequently are most suited for an analytic analysis too. Effect of electrical fields on fracture behavior of PZT ceramics has been investigated by Suo[1]et.al. Park and Sun [2] proposed fracture criteria based on mechanical strain energy release rate, while Gao and co-workers [3] proposed a local energy release rate criterion for fracture of cracked piezoelectric plate. They proposed a sort of generalization of the classical Dugdale model for plastic yielding near crack tip in thin metal sheet of piezoelectric ceramic, which is electrically ductile and mechanically brittle. Based on Gao' model a modified Dugdale model is proposed for arrest of a crack weakening a piezoelectric poled plate.

CASE – I: CRACK LYING PERPENDICULAR TO THE POLING AXIS

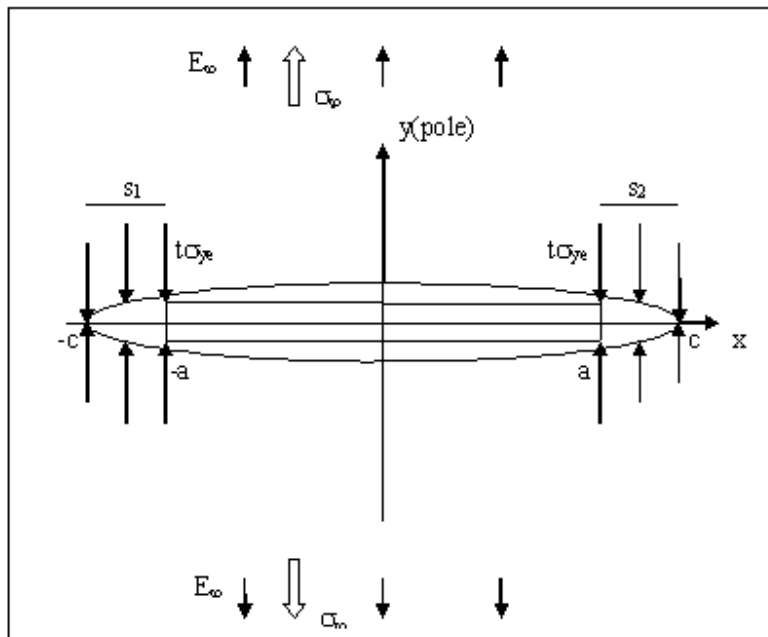


Figure 1: Schematic diagram of crack lying perpendicular to poling axis

An infinite transversely isotropic poled piezoelectric plate is weakened by a finite hairline straight crack lying perpendicular to the poling axis. Uniform constant tension and electric field applied remotely, opens the crack faces forming plastic and electrical zones ahead of each tip of the crack. The plate being mechanically brittle the mechanical stress singularity is encountered first. The mechanical plastic zones s_1 and s_2 develop at the tips $-a$ and a , respectively and occupy the region $[-c, -a]$ and $[a, c]$ on ox axis. Each of the plastic zones s_1 and s_2 is arrested by applying electrical displacement and linearly varying yield point stress at the rims of these plastic zones .

CASE – II: CRACK LYING ALONG THE POLING AXIS

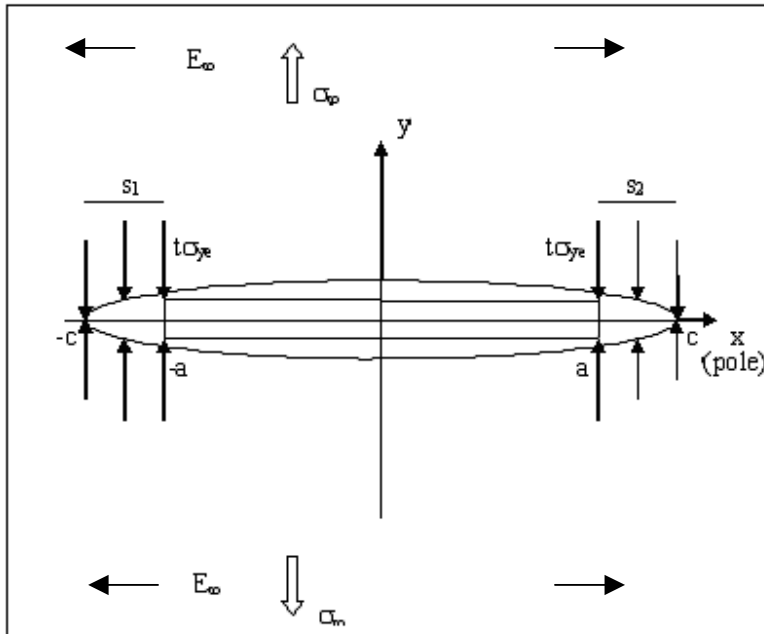


Figure 2: Schematic diagram of crack lying along the poling axis

An infinite poled piezoelectric plate weakened by a finite hairline straight crack lying along $[-a, a]$ on ox (which is also the direction of poling in this case). The uniform constant tension applied at infinite boundary opens the crack faces in Mode I type deformations. Consequently a plastic zone is formed ahead of each tip of the crack. The case under investigation is of small scale yielding consequently the developed plastic zones are assumed to be lying along the length of the crack. These developed plastic zones are denoted by s_1 and s_2 respectively and occupy the segments $[-c, -a]$ and $[a, c]$ on ox axis. The rims of the plastic zones are subjected to linearly varying yield point stress of the plate, to arrest the crack from further opening. In this case there is no electrical yielding because the electric displacement field is finite everywhere.

Each of the case is solved using Dugdale hypothesis in conjunction with complex variable techniques developed by Muskhelishvili [4]. Closed form analytic solutions are obtained in each case. It is further intended to calculate stress intensity factors, crack opening displacement at the crack tip and local energy release rate in each of the two cases.

References

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