

A Fractal Cohesive Crack Model

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When the fractal dimension of the crack D is allowed to vary from 1 to 2, a fractal crack changes from a smooth crack to a plane filling entity, which is more like a void than a crack. Significant variations from the classic solution are demonstrated as the singularity exponent α , entering in the near-tip stress field, $r^{-\alpha}$, sweeps the range (0,0.5). The fractal cohesive crack model described here is based on a simplifying assumption, according to which the original problem is approximated by considerations of a smooth crack embedded in the stress field generated by a fractal crack. The well-known concepts of the stress intensity factor and the Barenblatt cohesive modulus, which is a measure of material toughness, have been re-defined to accommodate the fractal view of fracture. Specifically, the cohesion modulus, in addition to its dependence on the distribution of the cohesion forces, is shown now to be a function of the 'degree of fractality', reflected by the fractal dimension D , or by the fractal roughness parameter, H . Two measures of the characteristic material length parameter have been suggested for solids weakened by presence of a fractal crack. For most fractal cracks, when D is not too close to 2, the characteristic length is chosen as the length of the cohesive zone, R . Above a certain threshold value of D , the root radius of the equivalent blunted crack (r) is suggested as the characteristic length parameter. The equivalent blunted crack is chosen by use of the Neuber stress magnification concept and the classic fracture mechanics equations for a crack with a finite root radius. The threshold for D is chosen as 1.846, which is 7.7% below the value of D corresponding to a 2D entity, a plane filling fractal crack, for which $D = 2$. In the limit case r turns out to be a fraction of the crack length. As the degree of fractality increases, the characteristic material length constant is shown to rapidly grow to the levels around three orders of magnitude higher than those predicted for the classic case. Such phenomenon may be helpful in explaining a distinct size-sensitivity of fracture processes in materials with cementitious bonding such as concrete and certain types of ceramics, where fractal cracks are commonly observed. It is an experimentally confirmed fact that in these materials the size of the process zone, which frequently is identified with the characteristic length, and the size of the end zone as modeled by the cohesive crack representation, frequently approach the size of the entire specimen used in a typical laboratory test.

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