SHPB TECHNIQUE FOR IDENTIFICATION OF COMPLEX MODULUS UNDER CONDITION OF NON-UNIFORM STRESS

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Summary An SHPB procedure for non-parametric identification of the complex modulus was developed without assuming equilibrium or axial uniformity of the states of stress and strain. It was applied to testing of polypropylene with bars made of polymethyl methacrylate and aluminium, and it was compared with the classical SHPB 1-wave procedure.

INTRODUCTION

SHPB (Split Hopkinson Pressure Bar) testing is used extensively for study of the constitutive properties of materials at high strain rates. For testing of metals, it is common to use elastic high-impedance steel bars. Recently, there has been increased interest in testing soft materials, which has led to the use of viscoelastic low-impedance polymeric bars. Classical SHPB testing has been reviewed by Gray III [1], and a survey of SHPB testing of soft materials has been provided by Gray III and Blumenthal [2].

For an SHPB test to be considered valid, it is required that several conditions be fulfilled in an approximate sense. Some of them concerning the specimen are: (a) Equilibrium should prevail, (b) the states of stress and strain should be uniform, (c) the state of stress should be uni-axial, and (d) the effects of friction at the bar-specimen interfaces should be negligible. Some of these conditions are in conflict as (a) requires short specimens (low aspect ratios), whereas (c) and (d) require long specimens (high aspect ratios).

Equilibrium and axial uniformity of stress can be assessed by comparing the stresses at the bar-specimen interfaces. Significant stress differences may prevail due to the effects of inertia and wave propagation, especially at an early stage [3]. Because of the time and strain (often more than 1 percent) required to establish equilibrium, it has been considered that the modulus of elasticity cannot be estimated from a classical SHPB test [1]. In specimens made of polymeric materials, which at small strains are characterised by the complex modulus, it is particularly difficult to achieve equilibrium because of the low wave speeds. A parametric identification procedure [4] has been developed which does not require equilibrium or axial uniformity of stress and strain.

The aim of this study was to develop an SHPB procedure for non-parametric identification of the complex modulus without assuming equilibrium or axial uniformity of stress and strain, apply it to the testing of a polymeric material with use of polymeric as well as metallic pressure bars, and compare it with the classical SHPB 1-wave procedure. The procedure should be useful for testing materials which do not allow attachment of strain gauges and/or fabrication of relatively long specimens. Other wave propagation methods for identification of complex modulus commonly require measurements on specimens with lengths of the order of 1 m [5].

METHOD

The identification problem considered was as follows: Given the cross-sectional area A_b, the complex modulus E_b and the density r_b of the bars; the cross-sectional area A, the density r and the length a of the specimen; the distances b_1 and b_2 from the bar-specimen interfaces to the sections of measurement, and the Fourier transforms $F\{e_{RM}\}$ and $F\{e_{TM}\}$ of the measured strains associated with the waves reflected from and transmitted through the specimen; find the complex modulus of the material of the specimen.

An equation, exact in the context of 1D wave propagation, was derived which gives an estimation the complex modulus E in terms of the measured strains associated with the reflected and transmitted waves. For $b_1=b_2$ this equation takes the form $(1/2)(Z/Z_b-Z_b/Z)\sinh(\gamma a)= F\{e_{RM}\}/F\{e_{TM}\}$, where $g=iw(r/E)^{1/2}$ is a wave propagation coefficient, $Z=A(Er)^{1/2}$ is the characteristic impedance of the specimen, and $Z_b=A_b(E_b r_b)^{1/2}$ is the characteristic impedance of the bars. An approximation E_0 of E, valid at low frequencies and low specimen-to-bar characteristic impedance ratios, was also derived. It corresponds to the classical SHPB 1-wave procedure. SHPB tests with pressure bars made of polymethyl methacrylate (PMMA) and aluminium (AL), and polypropylene (PP) specimens with diameter 20 mm and lengths 10, 20, 50 and 100 mm were carried out. The complex modulus of PP was estimated by E and E_0, and these estimations were compared with a result E_{ref} from [6] which was used as a reference.

RESULTS AND DISCUSSION

For the PMMA bars, there was a good general agreement between E and E_{ref} for the 10, 20 and 50 mm specimens. For the 100 mm specimen, the agreement was fair. The deviations were manifested mainly by oscillations in E. For the AL bars, there was a good agreement between E and E_{ref} for the 10 and 20 mm specimens. For the 50 mm specimen, there was a fair agreement, but with oscillations in E. For the 100 mm specimen, there were strong oscillations in E. For both bar materials, E_0 overestimates the magnitude of the complex modulus. This overestimation increases with frequency f, specimen length a and the magnitude $|Z/Z_b|$ of the specimen-to-bar characteristic impedance ratio (about 0.20 for PP/AL and 0.70 for PP/PMMA). The estimations E and E_0 for the complex modulus of PP versus frequency f obtained with PMMA and AL bars, and specimens of lengths 10 and 50 mm, are shown in Fig. 1.

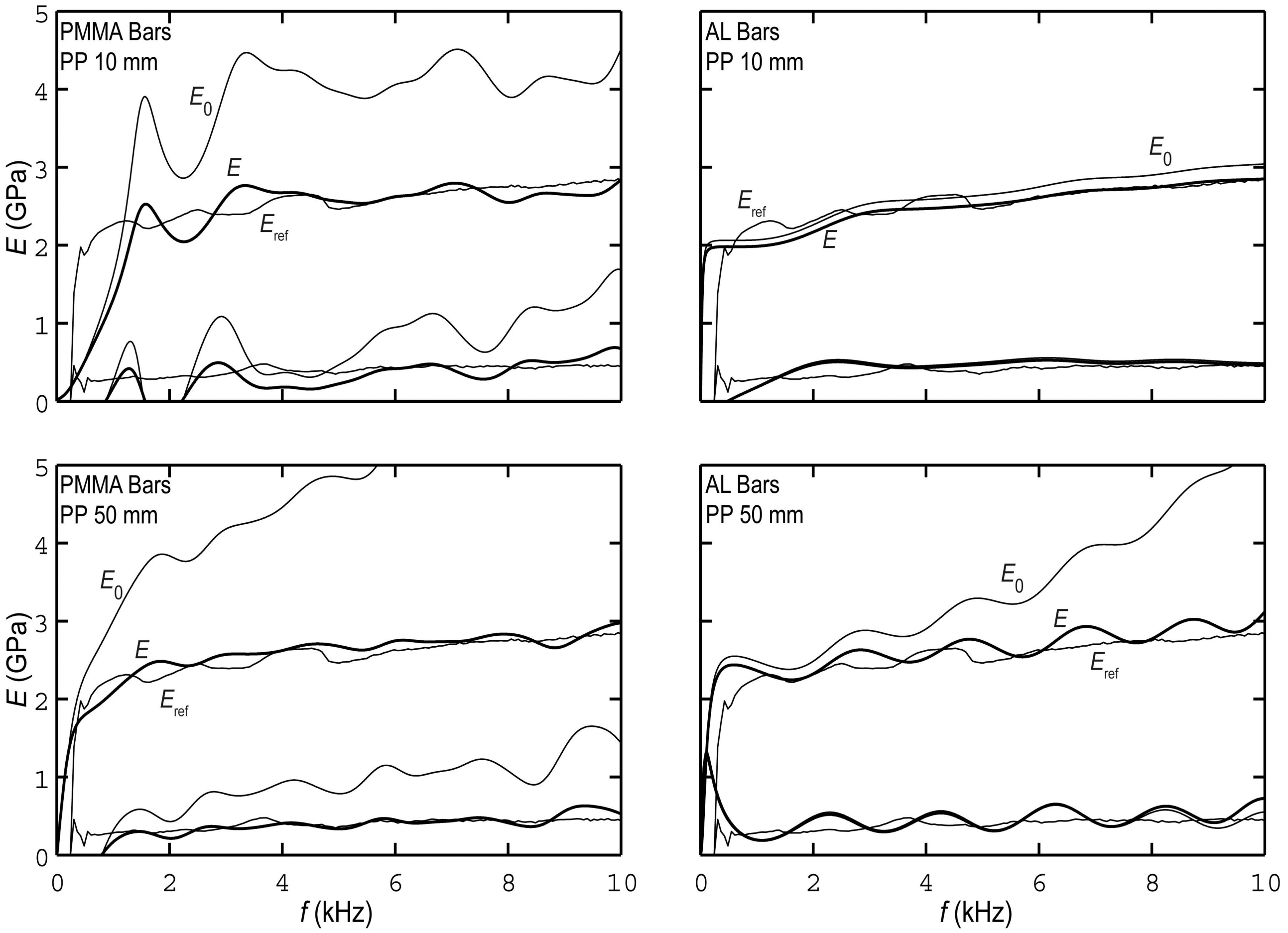

Fig. 1. Estimations E and E_0 of the complex modulus of PP versus frequency f obtained with PMMA and AL pressure bars, and specimens with diameter 20 mm and lengths 10 and 50 mm. The complex modulus E_{ref} of PP from [6] is shown as a reference. Real parts at top and imaginary parts at bottom of each diagram.

The oscillations in E were found to be sensitive to the time intervals chosen for isolation of the reflected and transmitted waves from other waves, which was done by cutting other waves and replacing them by zeroes. This indicates that the separation of the reflected and transmitted waves from other waves may not have been perfect as at very low signal levels it was not possible to distinguish contributions from the different waves, electrical disturbances and general noise. This explanation is supported by the circumstance that when AL bars were used, the oscillations in E increased with specimen length, i.e., with the lengths of the reflected and transmitted waves. For the PMMA bars, with shorter waves, the oscillations were relatively small and independent of specimen length. The oscillations were also found to be affected by the presence of even very weak bending waves in the pressure bars which may have disturbed the contact between the specimen and the pressure bars. Influence of 3D effects may also have had some effect on the oscillations.

ACKNOWLEDGEMENT

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