

ON CALCULATING EFFECTIVE ELASTIC PROPERTIES OF MEDIA WITH INCLUSIONS: ASYMPTOTIC REPRESENTATIONS AND AREAS OF THEIR APPLICABILITY

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The problem of calculating effective properties of heterogeneous media has been attracting a lot of attention; now a reader may find both original works on solving particular problems and comprehensive surveys. It is known that even a small amount of inclusions may result to significant change of the elastic moduli of composite. To estimate this contribution, among other, formulae for effective elastic moduli of media with isolated inhomogeneities are available for the following cases: media with flat disk-like inclusions of arbitrary moduli [1]; media with cracks, which may be considered as flat inclusions of vanishing moduli, eg, [2]; media with rigid flat inclusions [3]. The point is that these solutions do not much with each other: they may not be obtained one from another by limit transitions. Hence, one may conclude that either some of these solutions are not correct, or each solution is correct for its own restricted range of parameters, and these ranges do not overlap. To clarify the question of the area of applicability of these and other solutions is one of the points of the current work.

The central aim of the work is to obtain the exact and asymptotic formulae for calculating the effective elastic properties of media with isolated inhomogeneities (disperse composites) and to estimate the range of parameters of their applicability.

Rather general solution for calculating effective elastic moduli of the media with the small concentration of inclusions is constructed based on Eshelby's approach [4] for isolated inclusion in elastic media. In case of isotropic distribution of the inclusions the expressions for moduli are written in the *explicit form*. The constructed operative algorithm is based on calculating invariants of supplementary parts of effective tensor of elasticity rather than averaging contribution of the inclusions over all possible orientations.

A number of particular cases are then obtained by limit transitions. Namely, for disk-like inclusions three particular cases have been distinguished for various relations of two parameters – ratio of moduli of inclusions and matrix $\mathbf{m}^*/\mathbf{m}^0$, and aspect ratio b/a of inclusions:

- extremely rigid inclusions $\frac{\mathbf{m}^*}{\mathbf{m}^0} \gg \frac{a}{b} \gg 1$;
- extremely soft (crack-like) inclusions $\frac{\mathbf{m}^0}{\mathbf{m}^*} \ll \frac{b}{a} \ll 1$;
- inclusions of moderate rigidity $\frac{b}{a} \ll \frac{\mathbf{m}^*}{\mathbf{m}^0} \ll \frac{a}{b}$; $\frac{b}{a} \ll 1$.

The last two of the solutions coincide with the known ones, while the first one appears to be new (the known solution for these case [3], unfortunately, contain a mistake). The existence of various cases of such a kind was first pointed to in [3].

Similarly, by limit transitions from the general solution formulas were obtained for needle-like inclusions. Here two particular cases have been distinguished: inclusions of moderate and low rigidity, and extremely rigid inclusions. The former case correspond to solution obtained by Wu [1], the latter solution appears to be new.

Correspondingly, formulas were obtained for the arbitrary flat ellipsoidal inclusions. The same three cases as for disk-like inclusions have been distinguished. The solution for the case of cracks coincides with the well-known solution [5]. The solutions for the other two cases are new. It is interesting that in case of inclusions of moderate rigidity the obtained solution coincide with the solution for disk-like (circular) inclusions; therefore, the form of inclusions in plane is inessential in this case.

It is shown that for all cases of inclusions of moderate rigidity (and for soft needle-like inclusions) the change of elastic moduli is proportional to volumetric concentration of the inclusions. In case of extremely rigid and extremely soft flat inclusions the change of elastic moduli may be represented as approximately proportional to the power of 3/2 of the area of projection of the inclusion. In case of extremely rigid and needle-like inclusions the change of elastic moduli may be represented as proportional to the third power of the inclusion length. The last two cases may be accounted together by a statement that for them the change of elastic moduli is approximately proportional to the third power of the maximal dimension of the inclusion.

The theoretical estimations and numerical calculations were made to assess the areas of applicability of the asymptotics. It is shown that the applicability of each asymptotics is determined by relation between two parameters: ratio of moduli of inclusions and matrix $\mathbf{m}^*/\mathbf{m}^0$, and aspect ratio b/a of inclusions.

To estimate numerical values of these parameters the plots have been made showing isolines of mistakes given by asymptotic formulae to compare with the exact solution. Figure 1 represent such a plot for shier moduli of the composite with Poisson's ratios of matrix and inclusions being 0.45 and 0.25, respectively. These values of Poisson's ratios correspond approximately to the values typical for composites on the base of nanotubes and

nanoplates. Abscise axis corresponds to logarithm of ratio of moduli of inclusions and matrix m^*/m^0 . Ordinate axis corresponds to logarithm of aspect ratio b/a of inclusions. For other effective moduli and other values of Poisson's ratios the plots look similarly. The plot shows that using an inappropriate solution may lead to significant errors.

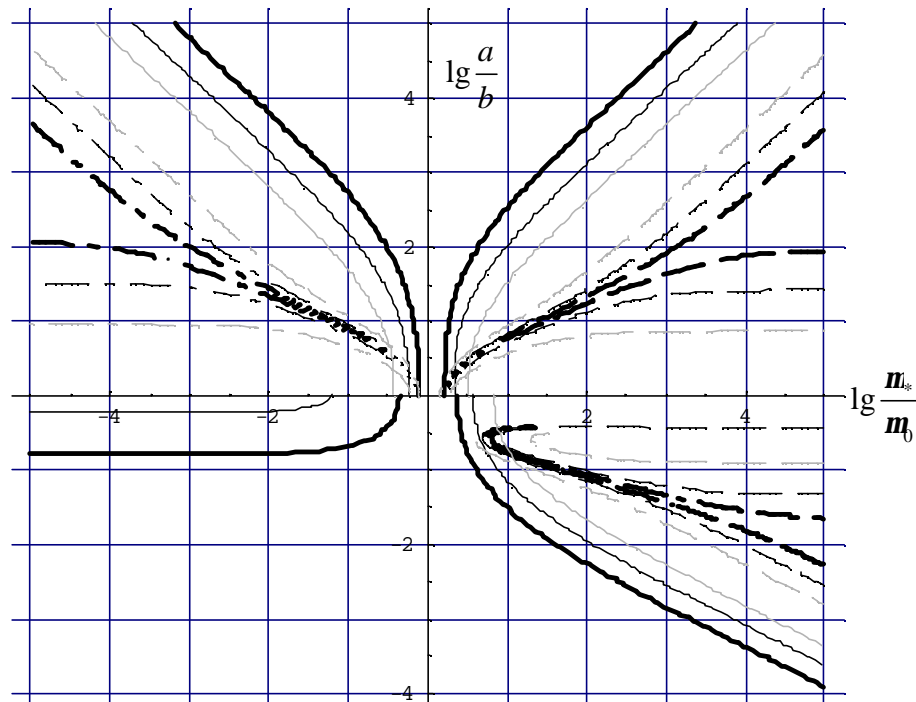


Figure 1. Area of applicability of asymptotic solutions. Solid lines – thin inclusions of moderate rigidity (upper half plane – disk-like, lower half plane – needle like); dashed line – extremely rigid inclusions (upper half plane – disk-like, lower half plane – needle like); dashed-dotted lines – extremely soft (crack-like) inclusions. Bold lines frame areas for which the corresponding asymptotics gives an error no more then 3%; thin black lines do for an error no more then 10%; thin gray lines do for an error no more then 30%

Literature

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