AN ASYMPTOTIC ANALYSIS OF MODE I CRACK IN CREEPNIG DAMAGED SOLIDS

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Summary To evaluate the mechanical behaviour around a Mode I crack tip the governing equations are formulated by light of Continuum Damage Mechanics. The asymptotic stress and continuity fields near the tip of a stationary crack are derived for non-linear viscous damaged materials, which deform according to the creep power constitutive law. The conventional Kachanov-Rabotnov creep – damage theory is utilized and the scalar continuity parameter is incorporated into the constitutive relations. Thus, the coupled system of damage mechanics – creep theory equations is considered. Based on the similarity variable a stress analysis is carried out for Mode I crack under plane stress and plane strain conditions assuming the existence of a totally damaged zone near the crack tip. It is found that the Hutchinson-Rice-Rosengren solution can't be used as the remote boundary condition and the actual far field stress is obtained. The shape of the totally damaged zone is given and analysed.

INTRODUCTION

Many works have been done to analyse the near crack tip fields coupled with elastic, elastic-plastic fatigue damage [1,2], creep damage [3,4] and elastic-plastic-viscoplastic damage. Some of the essential aspects of the considered set of problems and the results obtained can be highlighted. 1. The damage gives significant influence on the stress and strain (strain rate) fields near the crack tip. 2. The mathematical structure of governing equations is affected by the modelling of damage. 3. While the Hutchinson-Rice-Rosengren (HRR)–field of non-linear fracture mechanics always shows the stress singularity at the crack tip for any finite value of the stress exponent, the preceding material damage in front of the crack tip decreases the singularity, and may give non-singular stress field. 4. The totally damage and (or) active damage zone (process zone) need be modelled in the crack tip region.

In the present work the asymptotic stress, strain rate and continuity fields in the vicinity of the Mode I crack in damaged materials are obtained using the self-similar variable proposed by Riedel [5]. The form of the similarity solution has been introduced by Riedel. However, a general solution for the problem has not been constructed yet. The present contribution is an attempt to obtain the stress, strain rate and damage distribution near the crack tip as the dimensionless functions of the similarity variable under the assumption of the existence of a totally damaged zone near the crack tip. The shape and the characteristic length of the totally damaged zone are not known and should be found as a part of the solution. The asymptotic fields allow to find and to study the geometry of the totally damaged zone where the damage parameter reaches its critical value and all the stress tensor components are equal to zero.

FORMULATION OF THE PROBLEM AND BASIC EQUATIONS

Let us consider a semi-infinite crack in an infinite body in a material with constitutive equations formulated on the basis of the creep power law in the framework of Continuum Damage Mechanics.

$$\dot{\varepsilon}_{ij} = (3/2)B\left(\sigma_e/\psi\right)^{n-1} s_{ij}/\psi\,, \tag{1}$$

where ψ is the continuity (integrity) parameter, $\dot{\varepsilon}_{ij}$ is the creep strain rate, s_{ij} is the stress deviator, $\sigma_e = \sqrt{3s_{ij}s_{ij}/2}$ is the equivalent tensile stress, B is the creep coefficient and n is the creep exponent.

The cumulative damage evolution law can be described by the Kachanov-Rabotnov equation

$$\dot{\psi} = -A\left(\sigma_{eqv}/\psi\right)^m, \qquad \sigma_{eqv} = \alpha\sigma_I + \beta\sigma_e + (1-\alpha-\beta)\sigma\,, \tag{2}$$

where σ_I is the maximum principal stress, $\sigma = \sigma_{kk}$ is the hydrostatic stress, A, m are the material parameters.

Asymptotic remote boundary conditions are assumed to have the form

$$\sigma_{ij}(r \to \infty, \varphi, t) \to \tilde{C} r^s \bar{\sigma}_{ij}(\varphi, t)\,. \tag{3}$$

Note that if the far field stress is determined by the HRR-solution then $\tilde{C} = \left(C^*/(BI_n)\right)^{1/(n+1)}$ and $s = -1/(n+1)$. It implies that the stress field at infinity must approach the HRR-field. However it is shown that the remote boundary condition should be formulated in a more general form (3). The instantaneous response of a material characterized by equations (1) and (2) is non-linear viscous. The evolution of damage at short times after load application is concentrated in the vicinity of the crack tip and can be analysed under the remote boundary condition. If equation (3) represents the initial condition and the remote boundary condition, dimensional analysis shows that the damage mechanics equations must have similarity solution of the form

$$\sigma_{ij}(r,\varphi,t) = (At)^{-1/m}\Sigma_{ij}(R,\varphi), \quad \psi = \psi(R,\varphi), \quad R = r\tilde{C}^{1/s}(At)^{1/(sm)} \tag{4}$$

The dimensionless functions Σ_{ij} and ψ are as yet unknown. The validity of these solutions has to be verified by insertion equations (4) into the governing equations (material law, compatibility and equilibrium equations) and boundary conditions.

ASYMPTOTIC SOLUTION

The asymptotic expansions of the effective stress (the stress referred to the surface that really transmits the internal forces) and the integrity parameter for large distances $(R \to \infty)$ from the crack tip are supposed to have the form

$$\Sigma_{ij} / \psi(R,\varphi) = R^{s} f_{ij}^{(0)}(\varphi) + R^{s_1} f_{ij}^{(1)}(\varphi) + \ldots \quad (s, s_1 < 0), \quad \psi(R,\varphi) = 1 - R^{\gamma} g^{(0)}(\varphi) - R^{\gamma_1} g^{(1)}(\varphi) + \ldots \quad (\gamma, \gamma_1 < 0). \qquad (5)$$

The asymptotic expansions for the effective stresses and integrity parameter (5) are to be sought outside of the totally damaged zone where the traditional Continuum Mechanics governing system of equations is not valid. The system of non-linear ordinary differential equations for the dimensionless functions $f_{ij}^{(k)}(\varphi)$, $g^{(k)}(\varphi)$ is derived. The boundary value problem (the new eigenvalue problem for the eigenfunctions $f_{ij}^{(0)}(\varphi)$ and eigenvalue s) formulated is solved numerically using a Runge-Kutta-Fehlberg procedure and a shooting method. Numerical results are presented in Table:

		Plane strain		Plane stress	
n	m	s	$f''(0)$	s	$f''(0)$
1	1	-1,5	-0,75	-1,5	-0,75
2	$0,7n$	-1,0	-0,5	-1,1540	-0,5686
3	$0,7n$	-0,7716	-0,4372	-1,0	-0,5
4	$0,7n$	-0,6684	-0,4092	-0,9133	-0,4658
5	$0,7n$	-0,6179	-0,3985	-0,8580	-0,4428
6	$0,7n$	-0,5901	-0,3950	-0,8197	-0,4261
7	$0,7n$	-0,5732	-0,3943	-0,7919	-0,4134
8	$0,7n$	-0,5621	-0,3948	-0,7708	-0,4035
9	$0,7n$	-0,5543	-0,3958	-0,7543	-0,3955

Here $f(\varphi)$ is the angular part of the Airy stress function $F(r,\varphi) = r^{s+2} f(\varphi)$ and $f''(0)$ is its second derivative on the line extending the crack. It is found that $s_1 = s + sm$, $\gamma = sm$, $\gamma_1 = 2sm$ and the geometry of the totally damaged zone is given by the equation $\psi = 1 - r^{\gamma} g^{(0)}(\varphi) - r^{\gamma_1} g^{(1)}(\varphi) = 0$.

CONCLUSIONS

It can be concluded that the HRR-field does not govern the geometry of the totally damaged zone. The new asymptotic stress field at infinity is found. It is shown that the configuration of the totally damaged zone is entirely determined by the far field stress obtained. The geometry of the totally damaged zone in the vicinity of the crack tip for $n = 3$ is given in Figure 1.

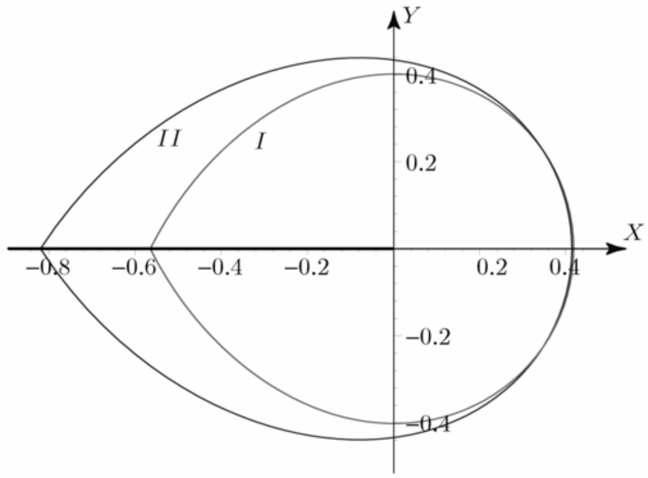

Figure 1. The geometry of the totally damaged zone in the crack tip region of a Mode I crack under plane strain conditions: I – the configuration given by the two-term asymptotic expansion of the continuity parameter, II – the configuration given by the three-term asymptotic expansion of the continuity parameter, $X = x\tilde{C}^{1/s}(At)^{1/(sm)}$, $Y = y\tilde{C}^{1/s}(At)^{1/(sm)}$.

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