DISCRETE PROBABILISTIC MODELLING OF DAMAGE AND ADHESION

Bernard Haussy [1], Jean François Ganghoffer [2]

[1] *École Supérieure d'Électronique de l'Ouest. 4, rue Merlet de la Boulaye, BP 30926, 49009 Angers Cedex, France*
[2] *LEMTA – ENSEM. 2, Avenue de la Forêt de Haye, BP 160, 54504 Vandoeuvre Cedex, France*

Summary An alternative to the continuous and deterministic modeling of continuum damage mechanics consists in a probabilistic approach: this is generally achieved for brittle materials by the weakest link model [1] and its generalizations, [2]. Discrete probabilistic models of interface damage are herewith elaborated, that extend the fibber bundle Daniel's model [3] to the consideration of different rupture probability laws and to a viscous behaviour of the interface.

We consider the following model system : a finite number N of parallel fibres are equally stretched between two rigid beams, representing a fibre bundle. The threshold traction force for the rupture of the fibbers is selected as a random variable. In order to emphasise the probabilistic aspects, the stiffness of all the fibres is taken equal to unity. The bundle is connected with a testing machine, considered as an elastic solid having a known stiffness k . The mean force, which is the applied force divided by the total number of fibbers N, is evaluated as $F(u) = (1 - P(u))u$, with $P(u)$ the cumulative distribution function. The displacement u of the interface varies between 0 – unloaded interface - to unity, which corresponds to the interface failure.

WEIBULL'S STATISTICS OF THE RUPTURE FORCES

For a distribution of the rupture threshold that obeys Weibull's law (Weibull's law depends on two parameters (α, β): for convenience and without restricting the generality, we choose $\beta = 1$ in the sequel), the total displacement U and the force F applied to the bundle express successively as

$$U = u + \frac{u e^{-(u/\beta)^{\alpha}}}{k} \; ; \qquad F = u e^{-(u/\beta)^{\alpha}}$$

The interface response is shown below (Fig. 1), in the case of a discrete behaviour (with a realization of 1000 fibbers).

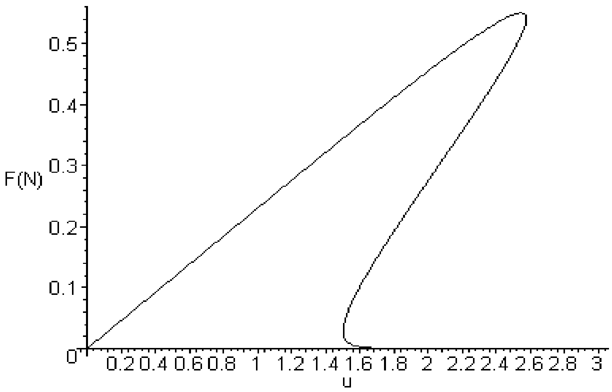

Fig. 1 : Weibull's law – continuous case - $\alpha = 4$

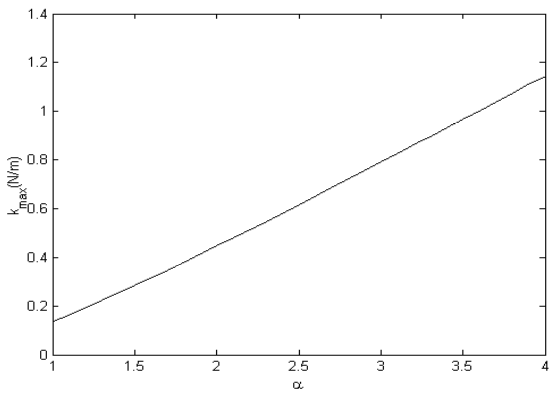

Fig. 2 : Maximum stiffness for bifurcation

A bifurcation is obtained when the maximum displacement is reached, $\frac{\partial U}{\partial u} = 0$. The condition for the existence of a bifurcation point thus resumes in finding the conditions to have real roots of the following equation, in which u is the unknown:

$$\frac{\partial U}{\partial u} = 1 + \frac{1}{k} e^{-u^{\alpha}} (1 - \alpha u^{\alpha}) = 0$$

The bifurcation behaviour occurs if the stiffness k is less than the obtained limit value $k(\alpha)$ (Fig. 2).

VISCOUS BEHAVIOUR OF THE INTERFACE

A viscous behaviour of the interface is further considered : the rheology of the interface is described by a bundle made of viscoelastic fibres ; selecting a Kelvin Voigt scheme, the relationship between stress and strain is accordingly given by

$$\sigma = E\varepsilon + \eta \frac{d\varepsilon}{dt}$$

where E is the Young modulus and η the viscosity of the interface material. The damage variable D is defined as the proportion of broken fibbers, thus the effective stress acting on the interface

$$\sigma := \frac{\Sigma}{1-D}$$

with Σ is the nominal stress. Considering a creep problem, - Σ is a constant - , the strain function $\varepsilon(t)$ satisfies the differential equation:

$$\frac{dt}{d\varepsilon} = \eta\!\!\left/\!E\right.\!\left/\left[\frac{\Sigma}{E(1-D)} - \varepsilon\right]\right.$$

that can either be solved analytically (in some cases, e.g. continuous damage) or numerically, in the case of discrete damage or for more sophisticated probability laws. Let investigate the case of continuous damage with a damage law $D(\varepsilon) = \varepsilon$, corresponding to the uniform law. For $\frac{\Sigma}{E} > \frac{1}{4}$, we obtain curves (time evolution of the total strain) presenting an inflexion point (dotted lines on Fig. 3) ; after that stage, damage accelerates, the system is no longer stable, and the interface breaks.

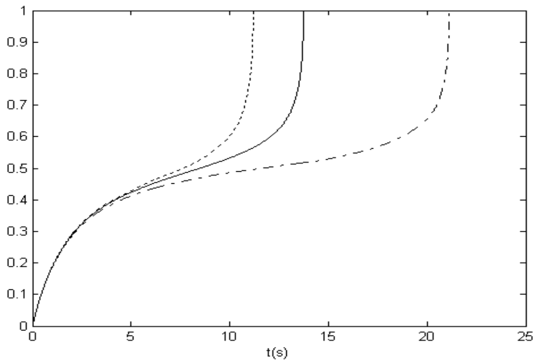

Fig. 3: Evolution of strain vs. time – Continuous damage (solid line) vs. discrete damage realizations (dotted lines)

If $\frac{\Sigma}{E} < \frac{1}{4}$, ε stays below 1 while reaching an asymptotic value. This modelling allows to take into account the different stages of creep: primary creep (no damage), secondary creep, ternary creep and finally break. Secondary and ternary creeps are due to the apparition of micro cracks, which is well described by damage. The comparison between continuous and discrete damage shows a real dispersion (Fig. 3) among the times before break, due to the probabilistic aspect of the modelling. Contrary to this, the case of a ram of applied stress minimizes the differences between different realizations of the probability laws.
This way of approaching the behaviour of interfaces has been extended to configurations representing adhesive bonding, and to the modelling of adhesion phenomena per se.

Amongst the perspectives, the importance of the time variable is one of the key aspect that shall further be investigated : in the probabilistic description of the behaviour of the interface, the role of time is equivalent to the role of strength in a traction test for example. Processes certainly have a non-markovian character: the whole history of the damage has then to be taken into account in order to estimate the evolution of the system.

References

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