# A MODEL FOR THE HYSTERESIS OF SHAPE MEMORY WIRES

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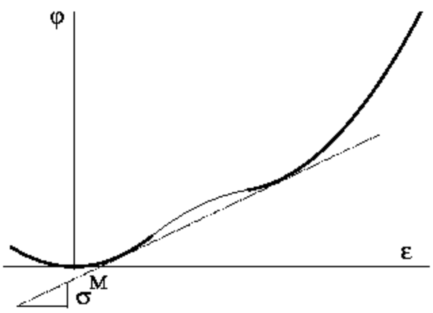
Summary We study the hysteretic behavior of shape memory wires under repetitive loading cycles. Our base assumption is a two well elastic energy that varies with the loading history to take into account the modifications of the material properties associated to the phase transition processes. The hysteretic behavior of this simple, predictive model, reproduces important phenomena observed in the cyclic experiments of shape memory wires.

## THE MODEL

The experiments on the mechanical behavior of shape memory wires subjected to cyclic loads (see *e.g.* [4], [5]) show that the hysteretic cycles become reproducible only after several "training" cycles: under successive cyclic loadings the stress-strain loops shift to lower values of the stress and decrease their size ("untrained regime") until a limit loop is achieved ("trained regime"). The description of the hysteretic behavior in the "untrained regime" is essential in the technological applications of such materials and is the subject of this paper. From a metallurgical point of view, the hysteretic behavior in the "untrained regime" can be associated to the complex mechanisms of defects formation and migration observed during the austenite-martensite phase transitions. Thus, the variation of the phase transition stresses may be associated to the processes of accumulation of dislocations and defects triggering the successive nucleation of martensite. Other macroscopic phenomena (see Fig.3) associated to the complicated microstucture evolution processes are the passage from constant stress *plateaux* to sloped transition paths and the presence of irreversible deformations that are associated to the effect of confinement of martensitic nuclei by microstructural defects.

In recent years, the constitutive hypothesis of non (quasi)convex elastic energies has been essential in a series of models for the hysteresis of materials undergoing martensitic phase transitions. Our main assumption is an energy with two wells (corresponding to the austenitic and martensitic phase) separated by a spinodal region where the energy is concave (see Fig.1). We suppose that the elastic energy $\varphi(x,\varepsilon(x))$ of each section $x \in (0,L)$ (here $L$ is the reference length and $\varepsilon$ is the strain) may change with the loading history and in particular with the number of preceding phase transitions at $x$. More specifically, we suppose that the limit stresses $s^+$ and $s^-$ (see Fig.1), defining the domain of stability of austenitic and martensitic phases, are assigned by two damage functions $s^+ = \hat{s}^+(x,n(x))$ and $s^- = \hat{s}^-(x,n(x))$, where $n(x)$ represents the number of times the section $x$ underwent a complete transition cycle austenite→martensite→ austenite.



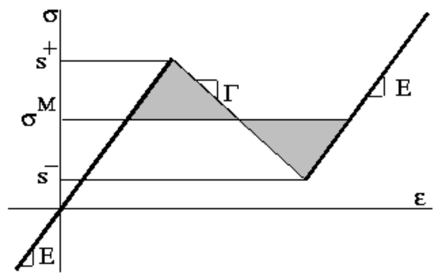




Fig. 1 Tri-parabolic energy and stress-strain relation. The Maxwell stress $\sigma^M$, that selects two equal areas in the stress strain diagram, assigns the energy difference between the wells.

If $\bar{\sigma}$ represents the assigned stress, the equilibrium solutions are the ones with a constant stress in the bar ($\varphi'(\varepsilon(x)) = \bar{\sigma}$, a.e. $x \in (0,L)$). Stability considerations (see [2,3]) exclude solutions with part of the bar in the spinodal region. Moreover, in accordance with the experimental evidences (see *e.g.* [4]), we may suppose that there exists one single interface between the austenitic and the martensitic phase. Thus we consider only the equilibrium solutions with the bar decomposed into two complementary sets: austenitic set $I_a$ and martensitic set $I_m$ (with $|I_a| + |I_m| = L$). Finally, to describe the evolution of the system under a generic loading history, we assume in accordance with the so called "maximum delay convention" (see [3]), that the system does not change its phase configuration, following the associated equilibrium branch, until it becomes unstable. In our case the stability domain of a given phase configuration is assigned by the austenitic section with the minimum value of the limit stress $s^+$ and by the martensitic section with the maximum value of the limit stress $s^-$.

## HOMOGENEOUS CASE

In Fig.2 we reproduce the hysteretic behavior of the described system under the hypothesis of homogeneous damage functions $s^+(x) = \bar{s}^+(n(x))$ and $s^-(x) = \bar{s}^-(n(x))$. As the figure shows, the system well reproduces the experimental evidences. In particular, observe that according with the experiments (*e.g.* [4]), in the first cycle, when the material is in its undamaged state, the transition corresponds to constant stress *plateaux*. On the contrary, if the system is subjected to successive cycles, due to the damage associated to preceding transitions, the system changes phase following multi-*plateaux* transition paths (with different values of the transition stress).

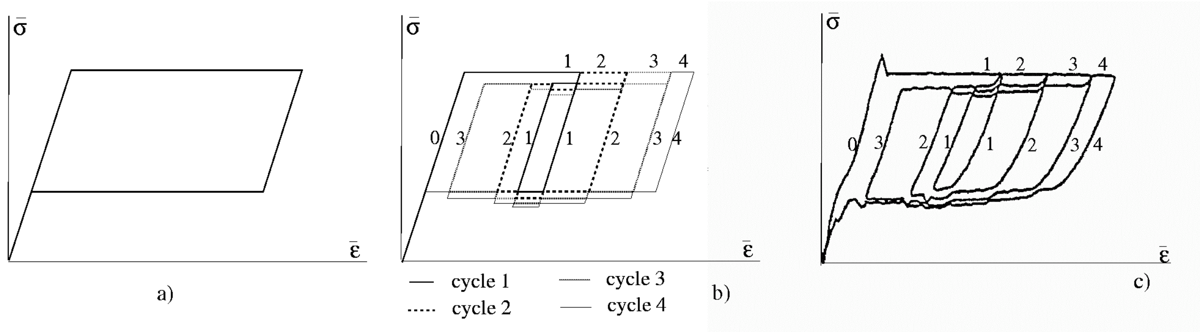



Fig. 2. a) Hysteretic behavior of the system starting from the virgin state $n(x) = 0, x \in (0, L)$ . b,c) Hysteresis under successive cycles of increasing amplitude: b) model, c) experiments (reproduced from [4]).

## INHOMOGENEOUS CASE

In Fig.3 we represent the hysteretic behavior in the second case when the damage functions are inhomogeneous. In particular for each given values of $n$, we assume gaussian distributions of the parameters $s^+$ and $s^-$. The model is predictive and the system reproduces both the described effects of the transition from constant transition *plateaux* to sloped paths and the insurgence of residual strains.

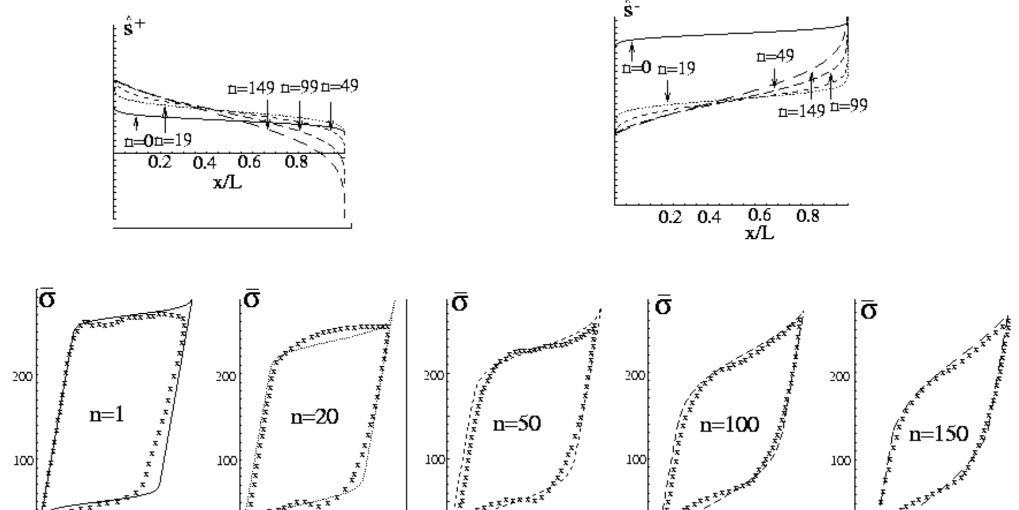



Fig. 3. a) Hysteretic behavior for the system in the hypothesis of non-homogeneous damage functions. In the upper figures we represent the functions $s^+$ and $s^-$ considered at the different value of $n$. In the bottom figures we compare the theoretical results with the experiments that we reproduced from [5].

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### References


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