Slow Snake-like Motions of Linkages.

T.Yu. Figurina
Institute for Problems in Mechanics of the Russian Academy of Sciences
Moscow, Russia

We investigate the possibility of slow (quasi-static) motion of linkages along a horizontal plane owing to changing their configurations. It has been shown in [1] that the quasi-static motion of a two-member linkage, occurring when the angle between its links varies, is uncontrollable and that the trajectories of the system's vertices are uniquely defined by the initial position of the system. On the other hand, it has been shown in [2] and [3] that both two-member and three-member linkages are controllable (i.e., they can be driven to a prescribed position on the plane), when using both fast and slow motions.

We state here quasi-static controllability of three-member linkages, both star-like and with links connected in series, as well as that of two-member linkages with variable lengths of the links. All considered linkages lie on a horizontal plane with dry friction. Their links are supposed to be weightless, and point masses m_i are located at the vertices A_i. We consider quasi-static (slow) motions of the linkages i.e., the motions with infinitesimal velocity and acceleration. At any time instant, control torques (forces) are chosen in order to satisfy the equilibrium conditions for each link of the system. A necessary equilibrium condition for each linkage is expressed as follows:

$$\sum_i \mathbf{F}_i = 0 \qquad \sum_i M_i = 0 \tag{1}$$

where $\mathbf{F}_i$ is dry friction force acting at the vertex A_i and M_i is its moment. If this condition is satisfied, then we can uniquely choose the control torques (forces) such that the equilibrium condition for each link of the system is also satisfied.

Star-like three-member linkage. Consider a star-like three-member linkage consisting of three identical rigid links A_0A_i, $i = 1, 2, 3$, connected by a joint with two motors generating the control torques acting between the links A_0A_3 and A_0A_i, $i = 1, 2$. All end point masses are the same, $m_i = m$.

Proposition 1. If $\frac{m_0}{m} \geq 2$, then the quasi-static motion of the star-like three-member linkage with moving central vertex is impossible. If $\frac{m_0}{m} < 2$, then the star-like three-member linkage is quasi-statically controllable.

We construct a number of gaits allowing the linkage to be driven to a prescribed position on the plane. The gaits depend on the parameter $\frac{m_0}{m}$. Figure 1 illustrates the motion of the linkage with $\frac{m_0}{m} = \sqrt{3}$.

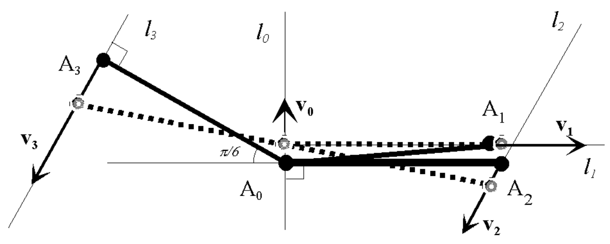

Fig.1.

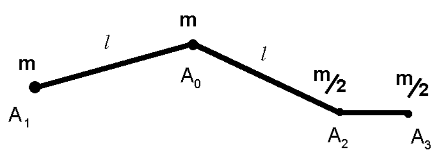

Fig.2

Three-member linkage with links connected in series. Consider a three-member linkage with rigid links connected in series, with two motors located at the inner vertices (Fig.2). The lengths of two links are equal to l and one end link is much shorter then the other links. The vertices of this end link carry equal masses, of magnitude $m/2$ each, and the other two point masses are equal to m. This linkage can be treated as a two-member linkage with equal masses located at all vertices. The length of one link of this linkage is fixed, whereas the length of the other link is variable and can take on one of the values $(1 + \epsilon)l$ or $(1 - \epsilon)l$.

Proposition 2. The three-member linkage with links connected in series is quasi-statically controllable, and, moreover, the vertex A_0 can be driven along a prescribed broken line on the plane.

Two-member linkage with variable lengths of the links. Consider a two-member linkage with variable lengths of the links, controlled by the forces acting along the links, and with fixed angle 2α between the links. We suppose that the end point masses are equal, $m_1 = m_2 = m$, and $m_0 \in m[\sin 2\alpha, 2\min\{\sin\alpha, \cos\alpha\}]$. Then one can quasi-statically drive the linkage to a prescribed position on the plane. To rotate a linkage one should elongate one link and shorten the other link simultaneously, and to shift a linkage one should elongate the links consecutively (central vertex is fixed) and then shorten both of the links simultaneously (central vertex moves). If we can control the angle between the links, then the linkage is controllable when $m_0 \in (0, 2m)$.

Motion of a homogeneous snake Consider finally a homogeneous snake which can bend at each point of its body. The snake can be modelled by a linkage with large enough number of identical rigid links. We investigate the gaits of the snake. Figure 3 illustrates the efficient gait enabling the snake to move quasi-statically along a given direction.

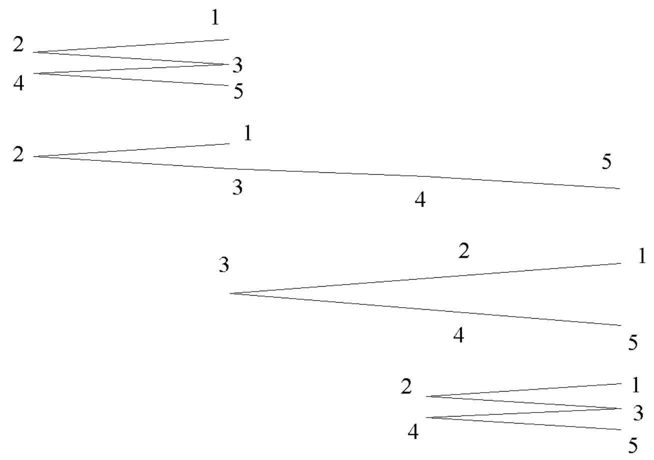

Fig.3

At any time instant, all moving points of the snake have the same velocities.

[1] Figurina, T. Yu., 2001, Quasistatic motion of two-link system along the horizontal plane, *Proceedings of CLAWAR*, Fourth Int. Conf. on Climbing and Walking Robots, 497–504.

[2] Chernousko, F.L., 2001, Controllable motions of a two-link mechanism along a horizontal plane, *Journal of Applied Mathematics and Mechanics*, Vol.65, No.4, pp.565-577

[3] Chernousko, F.L., 2000, The motion of a multilink system along a horizontal plane, *Journal of Applied Mathematics and Mechanics*, Vol.64, No.1, pp.5-15