SURFACE ACCURACY OF INFLATABLE REFLECTOR COVERED WITH STRETCHED CABLE

Kazuo TANIZAWA*, Jun NISHIMURA** and Kazuo Yamamoto*
* *Department of Intelligent Systems, Kinki University, 930 Nishimitani, Uchida, Nagagun 649-6493, Japan*
** *The Institute of Space and Astronautical Science, JAXA, 3-1-1 Yoshinodai, Sagamihara 229-8510, Japan*

Summary We present here an idea of the inflatable reflector and the surface error evaluation of this reflector, which is segmented into N-regular polygonal facets by the uniformly stretched cable net to improve the reliability on surface accuracy. From this study, we made it clear the effects of design parameters on the surface accuracy, such as the internal pressure, the cable tension, the membrane tension, the facet size, and etc. We also derived simple approximate formulas to give the RMS surface accuracy within 5% error in the realistic design region, where the surface error was almost proportional to the ratio of the cable force to the membrane tension.

INTRODUCTION

Although the inflatable reflector has a long history of development, it has not yet been put to practical space use, and only few papers related to the design of accurate surface have been presented[1)]. The inflatable reflector is largely behind to the competitive mesh reflector on the study and realization. In case of the inflatable reflectors there exist basic difficulties to form the accurate parabolic surface,because of deterioration of surface accuracy caused by inhomogeneity of material, folding lines in stowed configuration and the low performance of deployment repeatability. The surface accuracy of the conventional inflatables is highly sensitive to the non-uniformity of the material and internal pressure.

To achieve the accurate shape of surface, we propose the inflatable reflector which is covered by the inextensional N-regular polygonal supporting cable net, and show its practical structural validity using the detailed surface deformation analysis with the relevant surface error estimation. In this structure, the macroscopic configuration is determined by the stretched supporting cable net which is not so sensitive to the tension or the internal pressure and the smooth shape of each facet is formed by the flexible membrane. Since effects of the inhomogeneity of material and the change of the internal pressure on the surface accuracy are restricted within each facet, this type of structure is able to easily attain high and stable surface accuracy.

As the above results needs the complicated calculations, it is somewhat tedious to treat the problem in a practical sense. In this paper, we present a simple approximate treatment to estimate the surface error of this reflector. It is found that the surface error of the objective surface is approximately proportional to the ratio of the cable force to the membrane tension, and these approximate formulas give the results within an accuracy of 5% error in the realistic design region.

ANALYSIS

We consider the case that the additional tension of the membrane due to its deflection in the equilibrium state is negligibly small compared with the initial tension of the membrane. It is also assumed that the each facet portion of the parabolic surface is approximated by the spherical shape as shown in Fig.1. The N-regular polygonal facets with the side length L are stretched under uniform tension N_m on the sphere surface of the curvature ρ. Each facet is bounded by the supporting cables with the uniform tension T_s. The rectangular coordinates X, Y and Z is defined as shown in Fig.1. Here, one-2Nth part of the facet ($0 \leqq Y \leqq X\tan\phi$ and $0 \leqq X \leqq R\cos\phi$) is considered from the N-symmetric feature of the facet deformation.

When the displacement W in the Z direction is sufficiently small to the side length L, the equilibrium conditions in the facet membrane, on the cables (so called self-consistent equilibrium), and on the each lattice point are described as (for details see literature[2)]),

$\partial^2 W/\partial X^2 + \partial^2 W/\partial Y^2 = p/N_m$ in facet membrane (1)

$T_s\, \partial^2 W/\partial Y^2 = 2N_m(\partial W/\partial X - \theta)$ at $X = R\cos\phi$ (2)

$\partial W/\partial Y = \partial W/\partial X = \theta\tan\phi$ at $(X,Y) = R(\cos\phi, \sin\phi)$ (3)

$W=0$ at $(X,Y) = R(\cos\phi, \sin\phi)$ (4)

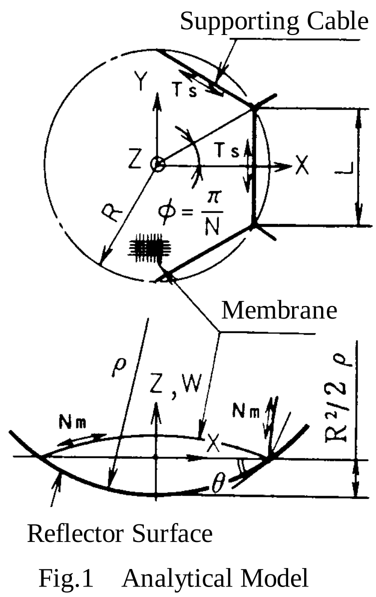

Fig.1 Analytical Model

Considering that the regular function of complex variable z(=x+jy; x=X/R, y=Y/R) satisfies the Laplace equation,and a special solution can be expressed by the quadratic expression of x and y, we get the general solution of the equation (1) expressed by the power series expansion of the complex variable as follows;

$$\frac{2\rho}{R^2} W = \mathrm{Re}\left(\sum_{i=0}^{\infty} A_i\, z^{N*i}\right) - \left(\frac{pL}{4N_m\theta}\right)(1 - x^2 - y^2)\cot\phi\ , \qquad (5)$$

where the first term of the right side means a pillow deformation W_p, the second term means a particular solution of the equation (1) W_s, and the normalization factor $R^2/(2\rho)$ denotes the maximum distance to reflector surface from the facet surface. Putting the equation(5) into the equations(2) ~(4) and arranging the equations with the new parameters that are the ratio of tension α (=N_mL/T_s) and a normalized pressure S_p(=$pL/(4N_m\theta)$), we obtain the following basic equations;

$$\sum_{i=0}^{\infty}(-1)^i A_i = 0 \quad (6) \qquad S_p = \tan\phi[1-(\frac{N}{2})\sum_{i=1}^{\infty}(-1)^i A_i] \quad (7)$$

$$\mathrm{Re}\sum_{i=1}^{\infty}[(\cos\phi + jy)^{N*i} - 2(N*i-1)\tan\phi + (\cos\phi + jy)^{N*i-1}\alpha/\cos\phi]A_i = 2\alpha + 2S_p(1-\alpha\cot\phi). \quad (8)$$

Since the equation (8) should be satisfied to arbitrary values of y, the equations to obtain the value of A_i are given by

$$\sum_{i=s}^{\infty}(N*i)!*(\cos\phi)^{N*i-2k}[\frac{\tan\phi}{(N*i-2k)!}+\frac{\alpha}{(N*i-2k+1)!}]A_i = 2\alpha + 2S_p(1-\alpha\cot\phi) \text{ at } k=1, \ =0 \text{ at } k>1, \quad (9)$$

where s means the minimum positive integer giving the positive number to the factorials of the equation. The equations (6),(7) and (9) are the final simultaneous equations to the deformation of the inflatable reflector.

NUMERICAL RESULTS

Best-fit Surface Error The best-fit surface error δ_{RMS} is defined by the following equation;

$$\frac{2\rho}{R^2}\delta_{RMS} = [\frac{1}{\Omega}\iint(W_1+W_2)^2 dXdY - \{\frac{1}{\Omega}\iint(W_1+W_2)dXdY\}^2]^{1/2}, \qquad (10)$$

where Ω, W_1 and W_2 are the area of facet, a membrane deflection(=W_p+W_s) and an objective parabolic surface given by W_2=$[R^2-X^2-Y^2]/(2\rho)$, respectively.

The numerical values of δ_{RMS} calculated from the equations (6),(7) and (9) with sufficient numbers of A_i ($i \leqq 20$) are shown on the logarithmic graph of Fig.2. Here, the data symbolized by ▲, ■ and ● mean the surface error δ_{RMS} of the triangular facet, the rectangular facet and the hexagonal facet, respectively. From this figure, it is found that δ_{RMS} is almost proportional to $(1/\alpha)$.

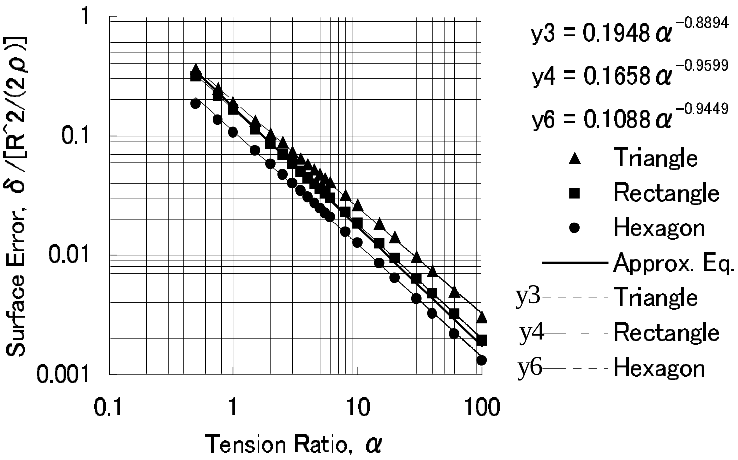

Fig.2 Effects of Tension Ratio α on Surface Error δ

Approximate design formula

Although the value of S_p should be determined rigorously from the force balance equation (7), it might be approximated well by the macroscopic force balance equation as follows;

$$S_p = \tan\varphi + (\tan^2\varphi)/\alpha \qquad (10)$$

Numerical results in Fig.3 show the validity of this approximation in the range of α >0.7. If we use the Eq.(11) instead of Eq.(7) and the approximate equation obtained in the mesh pillow deformation analysis[3),4)], we obtain the approximate design formulas on the surface error of the objective inflatable reflector which are shown as the broken lines in Fig.1.

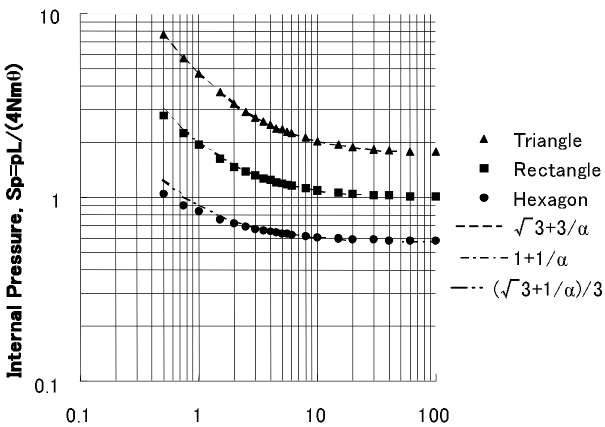

Fig.3 Relation between Tension Ratio α and Normalized Internal Pressure Sp

CONCLUSION

We proposed the inflatable reflector covered with the uniformly stretched cable net, and made clear the fundamental effects of design parameters on the surface error. We also presented the approximate formulas to evaluate the RMS surface error.

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