Viscoelastic Composites with Unbounded Overall Stiffness and Damping

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Summary Theoretical and experimental investigations on mechanical systems with negative stiffness inclusions revealed that composite materials may exhibit extreme high stiffness and damping. The causal mechanism is a greater deformation in and near the inclusions than the composite as a whole. Stability of this sort of systems is emphasized. We have shown that a negative stiffness element embedded in a positive stiffness spring network can be stable for extreme damping and metastable for extreme high stiffness.

COMPOSITE THEORY

Composite materials exhibiting extreme mechanical properties due to negative stiffness elements have been reported in several theoretical and experimental studies [1,2,3,4]. The rationale for expecting extreme behavior in systems with one negative stiffness phase can be understood by considering the following thought experiments based on simple mechanical spring models. For a series elastic system, $E_c = 1/(E_1^{-1} + E_2^{-1})$, where E_c, E_1, and E_2 refer to the Young's modulus of the composite, phase 1, and phase 2, respectively. V_1 and V_2 refer to the volume fraction of phase 1 and phase 2 with $V_1 + V_2 = 1$. The system of springs in series is analogous to the Reuss model in composites. Furthermore, we can express the compliance $j = 1/E$ as $j_c = j_1 + j_2$. If one stiffness is negative, the corresponding compliance is negative. We can sum a positive and negative compliance to obtain a zero compliance and hence an infinite stiffness. One can generalize above argument by using Hashin-Shtrikman formula [5] for estimating isotropic and homogenous two-phase composites. The bounds for isotropic composites are attainable, and hence are exact solutions for specific morphologies. Therefore assigning a negative value to one of the two constituents is legitimate. By a direct inspection, one can see singularities in the bounds when one of the phases' moduli is negative, which indicates extreme high overall shear modulus. With the aid of the elastic-viscoelastic correspondence principle, one is able to investigate dynamic modulus and linear viscoelastic damping. Relevant experimental observations can be found in [2,3]. The theoretical investigation of extreme coupled field quantities, such as thermal expansion coefficients and piezoelectric constants, can be found in [6].

STABILITY

The stability of systems with a negative stiffness element is intriguing. A bulk solid of negative stiffness materials is unquestionably unstable, but if all of its boundaries are fully constrained in displacement control, it can be stable, in which stability requires shear modulus (G) and Poisson's ratio (ν) to be $G > 0$ and $-\infty < \nu < 0.5$ or $1 < \nu < \infty$ [7]. Since Young's modulus is $E = 2G(1 + \nu)$ for isotropic homogeneous materials, the Young's modulus can legitimately be negative for constrained solids. Furthermore, if the condition of strong ellipticity, $G > 0$ and $\nu < 0.5$ or $\nu > 1$, is satisfied, isotropic and homogeneous materials can have negative bulk and Young's modulus without losing uniqueness in their elasticity solutions under no restrictions on boundary condition [8]. Also, it implies real waves can propagate in the media. For traction boundary value problems, satisfying the strong ellipticity condition does not imply stability. However, these criteria are only valid for homogeneous materials comprised of a single constituent. Lakes and Drugan [4] demonstrated that Reuss-type composites with negative stiffness inclusions are unstable, based on an energy argument in the linear elasticity context. A negative stiffness lumped element (a structure) may be stabilized by a hard constraint. Instead of investigating the stability problem through the viewpoint of continuum mechanics, we analyze the discrete rheological model, as shown in Figure 1, to understand the behavior of the system [9,10,11,12]. The model looks simplistic, but exhibits surprising results.

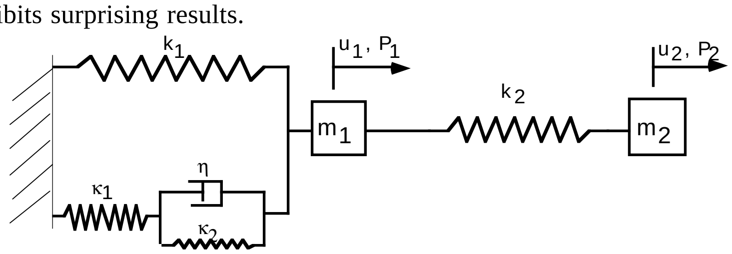

Figure 1. The rheological model with a negative stiffness element (κ_1).

The displacements and masses are denoted as u_1 and u_2, m_1 and m_2 at node 1 and 2, respectively. The symbols, k_1, k_2, κ_1, κ_2, denote spring constants, and η is viscosity. Following the Lyapunov indirect theorem [13], the eigenvalues of Jacobian matrix of the linearized dynamical system determine its infinitesimal stability. The numerical study of the system is shown in Figure 2 with the set of parameters, $m_1 = m_2 = 10^{-12}$ kg , $k_1 = 10$ kN/m, $k_2 = 5$ kN/m , $\kappa_2 = 5$

kN/m and η = 0.1 kN-s/m. If the real part of an eigenvalue of the coefficient matrix is greater than zero, the system is unstable. As can be seen, the damping, measured in terms of the loss tangent, which is defined as $\tan\delta \equiv \mathrm{Im}\{k^*\}/\mathrm{Re}\{k^*\}$, can approach infinity in the stable regime while overall stiffness is held finite. For a discrete spring element, complex force constant is denoted as k^*, and δ is the phase angle between sinusoidal force and displacement in time. The observability for extreme high stiffness and damping is discussed in Wang and Lakes [11].

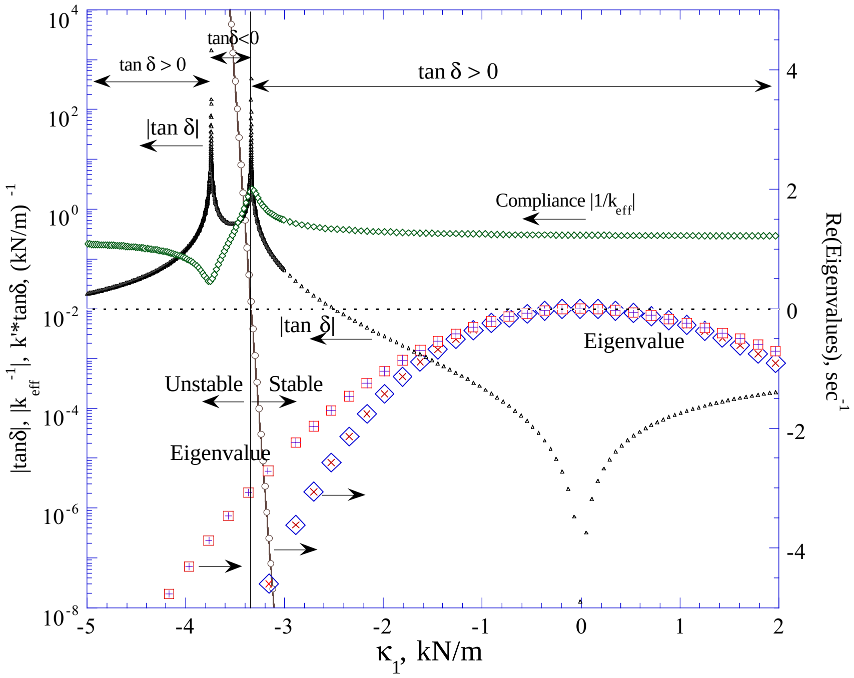

Figure 2. Stable high damping and anomalies in effective compliance in a discrete system with a negative stiffness element (κ_1), based on the model shown in Figure 1.

CONCLUSIONS

The possibility of constructing extreme high stiffness and high damping materials is presented. Stable high damping is achievable, and extreme high stiffness is metastable. With further considerations of the surface energy and particle size influence of negative stiffness inclusions, it might be possible to obtain composites with stable extreme overall stiffness.

References

1 Lakes, R. S., "Extreme damping in composite materials with a negative stiffness phase", *Physical Review Letters*, **86**, 2897-2900, 26 March 2001.

2 Lakes, R. S., "Extreme damping in compliant composites with a negative stiffness phase" *Philosophical Magazine Letters*, **81**, 95-100, 2001.

3 Lakes, R. S., T. Lee, A. Bersie, and Y. C. Wang, "Extreme damping in composite materials with negative stiffness inclusions", *Nature*, **410**, 565-567, 29 March 2001.

4 Lakes, R. S. and W. Drugan, "Dramatically stiffer elastic composite materials due to a negative stiffness phase?", *J. Mechanics and Physics of Solids*, 50, 979-1009, 2002.

5 Hashin, Z. and Shtrikman, S. A variational approach to the theory of the elastic behavior of multiphase materials. *J. Mech. Phys. Solids*, **11**, 127-140, 1963.

6 Wang, Y. C. and R. S. Lakes, "Extreme thermal expansion, piezoelectricity, and other coupled field properties in composites with a negative stiffness phase," J. Appl. Phys. **90**, 6458-6465, 2001.

7 Bramble, J. H. and Payne, L. E. On the uniqueness problem in the second boundary value problem in elasticity. *Proc. Fourth National Congress of Applied Mechanics*, 469-473, 1963.

8 Knowles, J. K. and Sternberg, E. On the failure of ellipticity and the emergence of discontinuous gradients in plane finite elastostatics. *J. Elasticity*, **8**, 329-379, 1978.

9 Wang, Y. C. and Lakes, R. S., Extreme stiffness systems due to negative stiffness elements, American Journal of Physics, in press, 2003.

10 Wang, Y. C. and Lakes, R. S., Stable extremely high-loss discrete viscoelastic systems due to negative stiffness elements, under review, 2003.

11 Wang, Y. C. and Lakes, R. S., Negative Stiffness Induced Extreme Viscoelastic Mechanical Properties: Stability and Dynamics, under review, 2003.

12 Wang, Y. C. and Lakes, R. S., Stability of negative stiffness viscoelastic systems, Quarterly of Applied Mathematics, accepted, 2003.

13 Leipholz, H., Stability of elastic systems, Sijthoff & Noordhoff, Alphen aan den Rijn, The Netherlands, 1980.