CREEP DAMAGE ANISOTROPY OF THINWALLED ELEMENTS STRUCTURES

Oleg K. Morachkovsky*, Vyacheslav N. Burlayenko**
*NTU "KhPI", Department of Theoretical Mechanics, 61002 Kharkov, Ukraine
**NTU "KhPI", Department of Applied Mathematics, Frunze 21, 61002 Kharkov, Ukraine

Summary. The paper is devoted to the development and theoretical justification of an anisotropy creep damage material model. For the description of initial anisotropy and damage-induced anisotropy the second-order damage tensor has been used. The numerical method of anisotropy creep damage lifetime prediction in thin-walled elements of structures was elaborated on the basis of the proposed model and FE scheme. The results of anisotropy creep damage analysis in models of plates were discussed.

ANISOTROPIC CREEP DAMAGE MODEL

In the presented work for the development of creep damage models using phenomenological approach the concept of the continuum mechanics has been used. For a mathematical description of this problem the three groups of equations were accepted. It is the balance equations, the thermodynamic laws and the constitutive equations. The unknown functions in them are the displacement vector of a particle **u** and her velocity vector, mass density $\rho(\mathbf{x},t)$, the Cauchy stress tensor $\boldsymbol{\sigma}$, the strain tensor $\boldsymbol{\varepsilon}$ and the deformation rate tensor. To simplify a *small deformation* are accepted. For general strain tensor at a creep the assumption of *additive nature* of reversible elastic **e**, responded Hook's law and irreversible creep **c** strain tensors was adopted: $\boldsymbol{\varepsilon} = \mathbf{e} + \mathbf{c}$, $\mathbf{e} = \overset{4}{\mathbf{E}} \cdot\cdot \boldsymbol{\sigma}$, where $\overset{4}{\mathbf{E}}$ is the fourth-order elasticity tensor.

Creep is an irreversible thermodynamical process and when formulating the creep laws it is necessary to meet the requirement of thermodynamically dissipative systems expressed by *Clausius–Duhem's* inequality. Then the additional unknown functions are appearing the internal energy, the heat flow vector **q**, the temperature T and the entropy s.

Relations between the independent variables of medium mathematically are represented by *constitutive equations*. The **creep model** was determined by following *state variables* are the deformation gradient or the elastic strain **e**, the temperature T and its gradient and *internal variables* associated with dissipative mechanism. In this case implying that damage evolution is predominant dissipative mechanism. For the description of damage-induced *anisotropy* in the consequence of creep a second-order damage tensor **d** has been assumed in form proposed by Murakami-Ohno [1].

According the assumptions concerning the constitutive equation the rate Helmholtz' free energy ψ is given by

$\dot{\psi} = \frac{\partial \psi}{\partial \mathbf{e}} \cdot\cdot \dot{\mathbf{e}} + \frac{\partial \psi}{\partial T}\dot{T} + \frac{\partial \psi}{\partial \mathbf{d}} \cdot\cdot \dot{\mathbf{d}}$, consequently the Clausius-Duhem inequality becomes:

$$\left(\boldsymbol{\sigma} - \rho \frac{\partial \psi}{\partial \mathbf{e}}\right) \cdot\cdot \dot{\mathbf{e}} - \rho\left(s + \frac{\partial \psi}{\partial T}\right)\frac{dT}{dt} - \rho \frac{\partial \psi}{\partial \mathbf{d}} \cdot\cdot \frac{d\mathbf{d}}{dt} + \boldsymbol{\sigma} \cdot\cdot \dot{\mathbf{c}} + \frac{1}{T}\nabla T \cdot \mathbf{q} \ge 0 .$$

Since for an isothermal process Clausius-Duhem's inequality can be written as the dissipation inequality. The additive decomposition of two dissipation mechanisms from *creep deformation* caused by the change of the dislocation structure and *damage* due to nucleation and growth of the microscopic cavities have been assumed: $\Phi = \boldsymbol{\sigma} \cdot\cdot \dot{\mathbf{c}} + \mathbf{R} \cdot\cdot \dot{\mathbf{d}} \ge 0$ $\Rightarrow \dot{D} = \boldsymbol{\sigma} \cdot\cdot \dot{\mathbf{c}} \ge 0$, $\dot{\Omega} = \mathbf{R} \cdot\cdot \dot{\mathbf{d}} \ge 0$, where $-\mathbf{R} = \rho \psi'_{\mathbf{d}}$ is the thermodynamic force associated with internal variable **d**.

In order to ensuring inequalities the existence of a scalar pseudo dissipation potential, a convex function of thermodynamic forces $\boldsymbol{\sigma}$ and **R**, using state variables **e**, **c** and **d** as parameters has been by postulated. In this case the thermodynamic potential was represented by two different positive convex functions: $F(\boldsymbol{\sigma},\mathbf{R};\mathbf{c},\mathbf{d},T) = F_c(\boldsymbol{\sigma};\mathbf{d},T) + F_d(\mathbf{R};\mathbf{d},T)$.

Arguments of introduced potentials $\boldsymbol{\sigma}$, **R** (**d**-variable) form potentials as invariants, which for anisotropic bodies will be mixed invariants of these tensors and material symmetry tensors, the kind of anisotropy:

$$F_c(\boldsymbol{\sigma};\mathbf{d},T) = (\sigma_e)^2,\ \sigma_e = \sigma_1 + \sigma_2,\ \sigma_1 = \boldsymbol{a} \cdot\cdot \boldsymbol{\sigma},\ \sigma_2^2 = \boldsymbol{\sigma} \cdot\cdot \overset{4}{\mathbf{b}} \cdot\cdot \boldsymbol{\sigma},$$

$$F_d(\mathbf{R};\mathbf{d},T) = (R_e)^2,\ R_e = R_1 + R_2,\ R_1 = \boldsymbol{g} \cdot\cdot \mathbf{R} \text{ and } R_2^2 = \mathbf{R} \cdot\cdot \overset{4}{\mathbf{h}} \cdot\cdot \mathbf{R} .$$

Applying a general normality rule to concretized potentials, we have obtained tensor-linear equations of state for initially anisotropic materials completely satisfying thermodynamical requirements

$$\dot{\mathbf{c}} = \frac{\dot{D}}{\sigma_e}\left(\boldsymbol{a} + \overset{4}{\boldsymbol{b}} \cdot\cdot \boldsymbol{\sigma}/\sigma_2\right) \text{ and } \dot{\mathbf{d}} = \frac{\dot{\Omega}}{R_e}\left(\boldsymbol{g} + \overset{4}{\boldsymbol{h}} \cdot\cdot \mathbf{R}/R_2\right)$$

For the combination of strains and the damage tensorial model an appropriate *equivalence stress* was defined by the quantitative damage measure as the ratio of specific energy of dissipation of damage Ω to its damage limiting value

$$\omega(t) = \Omega(t)/\Omega_*,\ \Omega(t) = \int_0^t \dot{\Omega} dt,\ \dot{\omega}(t) = \dot{\Omega}(t)/\dot{\Omega}_*,\ \omega(0) = 0 \text{ and } \omega(t_*) = 1,$$

where Ω_* - is the constant quantity for the material at the given temperature.

Then, the tensor **R** is coupled with stress tensor by the relationship: $\mathbf{R} = \boldsymbol{\sigma}/(1-\omega)$ and for damage tensor we have a kinetic equation fitting the tensor model of (*Lemaitre - Chaboche*, 1994) [2].

The functions $\dot{D}$, $\dot{\Omega}$ $\dot{\omega} = \dot{\Omega}/\Omega_*$ in constitutive equations must be identified from uniaxial creep tests. The good approximation of experimental data for metal materials is given by follows functions: $\dot{D} = (\tilde{\sigma}_e)^N$, $\dot{\Omega} = (\tilde{\sigma}_{*e})^K/(1-\omega)^L$, where $\tilde{\sigma}_e = \sigma_e/(1-\omega)$, $\tilde{\sigma}_{*e} = \sigma_{*e}/(1-\omega)$, $\sigma_{*e} = \sigma_{*1} + \sigma_{*2}$, $\sigma_{*1} = \boldsymbol{g}\cdot\cdot\boldsymbol{\sigma}$, $\sigma_{*2}^2 = \boldsymbol{\sigma}\cdot\cdot\overset{4}{\boldsymbol{h}}\cdot\cdot\boldsymbol{\sigma}$.
If damage response on tension and compression are not distinguished (***a*** = ***g*** =0), it for metallic materials can be written

$$\dot{\mathbf{c}} = \frac{(\sigma_2)^{N-2}}{(1-\omega)^N}\overset{4}{\boldsymbol{b}}\cdot\cdot\boldsymbol{\sigma},\ \dot{\mathbf{d}} = \overset{4}{\boldsymbol{h}}\cdot\cdot\boldsymbol{\sigma}\,\dot{\omega},\ \dot{\omega} = \frac{(\sigma_{*2})^{K-2}}{(1-\omega)^{K+L-1}}\ .$$

For the practical using of the anisotropic creep model it is necessary to define constants K, N and L and material tensors from experimental researches. Taking into account the assumption of properties of *some material symmetry*, we can reduce number of the constants. Further, *transversally-isotropic* materials were studied.
The *verification* of the proposed creep damage-induced anisotropy model on the base of comparison with experiments results for three types of constructive materials: composite alloy *Co-Cr-C* [3], aluminium alloy *D16-AT* and titanium alloy *BT1-0* [4] was performed. The calculated and experimental data quite accurately correspond to each other.

NUMERICAL MODELLINGOF THINWALLED STRUCTURES

For many engineering structures operating at elevated temperatures and under intensive loading the creep analysis can be performed using engineering models of thin plates and shallow shells. The *mathematical statement* of the creep damage problem for thin plates and shallow shells was done on the basis of the general theory of the three-dimensional initial-boundary creep problem with some assumptions concerning of thin shells theory. The constitutive equations of proposed early anisotropic (transversally-isotropic) creep damage model for a case of multiaxial stress state, which take place in shallow shells and plates, were generalized.
For solving the creep damage problem of shallow shells the *variational approach* was used. Assuming that a creep strains are known at each time step was formulated a variational functional given on displacements:

$$J = \int_S \left(b_{ijkl}\varepsilon_{ij}^m\delta\varepsilon_{ij}^m - d_{ijkl}\chi_{kl}\delta\chi_{ij}\right)dS - \int_S p\,\delta w dS - \underline{\int_S N_{ij}^C\delta\varepsilon_{ij}^m dS} - \underline{\int_S M_{ij}^C\,\delta\chi_{ij}\,dS}\ .$$

The underline terms are defined by irreversible creep strains. The solutions at the fixed time moment correspond to variational equation of the Lagrange type for this functional, which includes additional work of pseudo-forces. The solution of linear boundary-value problem as a minimum point of the variational functional, defined on kinematically suitable displacements, was performed by a finite element method at each time step. For the meshing we use the 3-node plate element with 18 degrees of freedom. It have the shape functions are linear relatively homogeneous coordinates for approximation the membrane displacements and cubic polynomials for deflection of a shell. This element is based on the Kirchhoff-Love type shell theory and contains 10 Gauss points in the plane and 9 integration points through the thickness. The constitutive equations in FE definitions are integrated by the predictor-corrector iteration scheme.
On the basis of the proposed numerical method the researches of the initial and damage-induced anisotropy of thin plates under creep conditions were performed. At first the *influence of preliminary creep* on change of primary properties of the material was studied. The theoretical results are compared with experimental data obtained by J.Betten et al. [6]. The agreement of the calculated and experimental data is quite satisfactory.
Further the influence of *damage-induce anisotropy in plates with hole* for a tension and bending loading in a plain of the plates under creep conditions were considered. With the purpose of the proof of existence for innate anisotropy of the materials of dependence of stresses redistribution character and time of initial rupture from a degree them innate anisotropy the numerical simulation for plates with a hole for three materials types are performed. Generally during the anisotropy creep both minimal and maximal the time initiation rupture can be demonstrated.

CONCLUSIONS

The development and theoretically to justify constitutive equations of anisotropic (the case of transversally-isotropic) creep damage model with an initial anisotropy have been performed. The transformation of the material model on the case of multiaxial stress state taking place in thin-walled structures with the using geometrically non-linear theory of thin shells has been made. The method on the basis FE-scheme and of the modified Lagrange's variational principal for anisotropic creep damage simulation in thin-walled structures has been elaborated.

References

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