

## Regenerative Tool Chatter Near a Codimension-2 Hopf Point

**Pankaj Wahi**, Anindya Chatterjee

*Mechanical Engineering, Indian Institute of Science, Bangalore, India*

Regenerative tool chatter can cause undesirable vibrations during machining. The corresponding mathematical model is a delay differential equation (DDE). These DDE models have potential codimension-2 (or double) Hopf bifurcation points where two pairs of pure imaginary roots coexist. We analyze such a model near a codimension-2 Hopf bifurcation point. The method of multiple scales (MMS) is used directly, bypassing a center-manifold reduction. Our choice of the associated small parameter for the analysis allows us to *not* treat the vibration amplitude as small. Damping is assumed to be small. Both sub- and supercritical Hopf bifurcations are observed, depending on how two key parameters vary near the double Hopf point. A reverse subcritical pitchfork bifurcation also occurs as parameters are varied. The analytical observations are also verified via direct numerics. The supercritical bifurcations observed here are important in the context of *control*: a minor excursion into the unstable region causes small amplitudes of vibration, leaving scope for returning to stable operation. In contrast, in the subcritical regime, any excursion into instability leads immediately to large amplitudes from which return to stable operation is problematic.

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