SHORT WAVE-LENGTH DYNAMICS OF A STRING ON ASYMMETRIC NONLINEAR SUPPORTS

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Summary We present the results of analytical and numerical study of short wave-length nonlinear localized excitations (envelope solitons) in a string with concentrated masses supported by asymmetric nonlinear anchor springs. In main asymptotic approximation the coupled Nonlinear Schrodinger Equations are obtained. We compare the analytical representation of localized solution with the numerical data. The role of envelope solitons in general nonlinear dynamics of the system under consideration is discussed.

Contrary to conventional approach to nonlinear dynamics of the string on symmetric nonlinear elastic foundation dealing with long wave-length vibrations, we present a study of short wave-length dynamics taking into account discreteness of elastic supports and asymmetry of their nonlinear characteristics.

The approach is based on the transition to complex representation of dynamical equations [1, 2] and use of multiple scale expansions. In our calculations the infinite weightless string with uniformly distributed concentrated masses is supposed to be supported by asymmetric nonlinear anchor springs, spatial distribution of masses being coincided with that for the springs. Potential energy of the springs is described by a fourth power polynomial. In main asymptotic approximation we obtain two continuous equations of motion with respect to complex combinations of the envelopes of displacements and velocities. In important particular cases these equations coincide with the system of coupled Nonlinear Schrodinger Equations (NSEs) and have localized soliton-like solutions. The comparison with numerical results is also made.

We start from infinite system of ordinary nonlinear equations describing dynamics of the string under consideration

$$\frac{d^2u_j}{d\tau^2} + 2\varepsilon^2\gamma\frac{du_j}{d\tau} + u_j + 4\varepsilon u_j^2 + 8\alpha\varepsilon^2 u_j^3 + 2\beta(2u_j - u_{j-1} - u_{j+1}) = 0\ ,$$

where u_j are displacements of masses, $-\infty < j < \infty$. The solution is looked for as a slow modulation of nonlinear normal mode [3] with minimal wave-length, taking into account asymmetry of potential energy relative to tension and compression. Such a slow modulation can be described by two partial nonlinear equations in dimensionless variables presenting averaged ($\tilde{V}$) and relative (V) characteristics of the cell consisting of two neighbor masses

$$\begin{aligned} &\frac{d^2V}{d\tau^2} + 2\varepsilon^2\gamma\frac{dV}{d\tau} + (1+4\beta)V + 4\varepsilon(V^2+\tilde{V}^2) + 8\alpha\varepsilon^2 V(V^2+3\tilde{V}^2) + 2\beta\varepsilon^2\frac{\partial^2 V}{\partial z^2} = 0\ ,\\ &\frac{d^2\tilde{V}}{d\tau^2} + 2\varepsilon^2\gamma\frac{d\tilde{V}}{d\tau} + \tilde{V} + 8\varepsilon V\tilde{V} + 8\alpha\varepsilon^2\tilde{V}(\tilde{V}^2+3V^2) - 2\beta\varepsilon^2\frac{\partial^2\tilde{V}}{\partial z^2} = 0\ . \end{aligned} \tag{1}$$

The spatial derivatives have coefficients of no less than second order of smallness. Small parameter ε characterizes a smallness of displacements as well as distance between neighbor masses (relative to wave-length of modulation) and friction. Using the following substitutions

$$\begin{aligned} \varphi &= e^{i\mu\tau}\left(\frac{dV}{d\tau} + i\mu V\right)\ ,\\ \tilde{\varphi} &= e^{i\tau}\left(\frac{d\tilde{V}}{d\tau} + i\tilde{V}\right)\ , \end{aligned} \tag{2}$$

where $\mu = (1+4\beta)^{1/2}$, $i = (-1)^{1/2}$ and introducing the slow times $\tau_1 = \varepsilon\tau_0$, $\tau_2 = \varepsilon^2\tau_0, \ldots$ alongside with 'fast' time τ_0, we look for the solutions as power expansions by parameter ε

$$\varphi = \varphi_0 + \varepsilon\varphi_1 + \varepsilon^2\varphi_2 + \ldots \tag{3}$$

In such a case differential operator can be expressed as

$$\frac{d}{d\tau} = \frac{\partial}{\partial\tau_0} + \varepsilon\frac{\partial}{\partial\tau_1} + \varepsilon^2\frac{\partial}{\partial\tau_2} + \ldots$$

After substitution of (3) in (1) and taking into account (2) we obtain in the main asymptotic approximation

$$\frac{\partial\varphi_0}{\partial\tau_2} + \gamma\varphi_0 + i\varphi_0(A_1|\varphi_0|^2 + A_2|\tilde{\varphi}_0|^2) + \frac{i\beta}{\mu}\frac{\partial^2\varphi_0}{\partial z^2} = 0\ ,$$

$$\frac{\partial\tilde{\varphi}_0}{\partial\tau_2} + \gamma\tilde{\varphi}_0 + i\tilde{\varphi}_0(A_3|\varphi_0|^2 + A_4|\tilde{\varphi}_0|^2) - i\beta\frac{\partial^2\tilde{\varphi}_0}{\partial z^2} = 0\ ,$$

Particular solution of the system is an envelope soliton (for $\gamma = 0$)

$$\varphi = Ce^{i(kz-\omega t)}\text{sech}[L(z - vt)] \ ,$$

$$\tilde{\varphi} = \tilde{C}e^{i(\tilde{k}z-\tilde{\omega} t)}\text{sech}[L(z - vt)] \ ,$$

In terms of initial variables such a soliton is an oscillating localized excitation which can propagate along the string. It is shown that existence of envelope solitons is determined by special relationship between parameters reflecting contributions of symmetric and asymmetric nonlinearities into potential energy.

Using analytical solution which is an approximate one for initial system we have performed a numerical study of stability of envelope solitons as well as their interaction (for $\gamma = 0$ and $\gamma \neq 0$). The diapason of energetic parameters providing elasticity of interactions is found. The role of such solutions in general nonlinear dynamics of the system under consideration is discussed. In particular we study manifestations of envelope solitons in the processes caused by excitations of rather complicated type including chaotic ones. It is shown that their role depends strongly on the intensity of excitations.

ACKNOWLEDGMENTS

This research was funded by the Russian Foundation for Basic Research (project 04-03-32119). This support is greatly appreciated.

References

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