INTERACTION OF PRIMARY AND INTERNAL 1:1 RESONANCES IN NONLINEAR SYMMETRIC 2DOF SYSTEMS WITH CUBIC NONLINEARITIES

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Summary. The primary resonance at forced oscillations in symmetric cubic two-degree-of-freedom-systems with close eigenfrequencies under harmonic excitation is studied. A complete analysis of the number, stability and configuration of coupled steady-state modes (CSM) in phase spaces is carried out. It is shown that in damped systems the CSM's are not exact normal or elliptic modes but they asymptotically approach these modes when the energy of oscillations increases.

Forced oscillations of nonlinear two-degree-of-freedom-systems with cubic nonlinearities having close natural frequencies are studied in a few papers ([1, 2] and others). It has been established that the internal resonance significantly affects the topology of the fundamental resonance curves. But the problem is rather complicated and is only slightly touched until recently. In particular, the bifurcation of only *normal* modes was taken into account. Such an approach is insufficient since steady–state motions include also *elliptic* modes (it has been shown yet for *autonomous* cubic systems with close eigenfrequencies), and all these modes and their bifurcations play a significant role in nonlinear behavior of systems under consideration.

The aim of this work is to carry out a complete analysis of interaction between the primary and internal 1:1 resonances in symmetric cubic systems under harmonic excitation with account of all steady-state modes..

Equations of motion are taken in the form (an external force acts only on the first degree of freedom and has order of a small parameter ε, damping coefficients are assumed to be equal for both degrees of freedom, they have order of ε, as well as nonlinear coefficients b_{ij} (*i, j*=1,2)):

$$\begin{aligned} &\ddot{u}_1 + 2\varepsilon\mu\,\dot{u}_2 + \omega_1^2 u_1 + \varepsilon\,(b_{11} u_1^3 + b_{12} u_1 u_2^2) = \varepsilon f \sin\Omega t \\ &\ddot{u}_2 + 2\varepsilon\mu\,\dot{u}_2 + \omega_2^2 u_2 + \varepsilon(b_{12} u_1^2 u_2 + b_{22} u_2^3) = 0 \end{aligned} \qquad (1)$$

where u_1, u_2 are principal coordinates of the linearized system.

We consider the case of closed natural frequencies, and frequency of the external force is also assumed to be close to the eigenfrequencies. So it is assumed that $\omega_2 \equiv \omega_1 + \varepsilon\sigma$, $\Omega = \omega_1 + \varepsilon\delta$, where σ and δ are detuning parameters.

Set of equations (1) is solved by the multiple scales method, with using the complex representation of hamiltonian equations corresponding to set (1). Introducing amplitudes a_k and phases θ_k of modes by expressions $u_k = a_k \cos(\Omega t - \theta_k)$, *k*=1,2, a following set of four first-order ordinary differential equations governing the amplitude–frequency modulation is derived (here $T_1 = \varepsilon t$ is the "slow time"):

$$\begin{aligned} &\frac{da_1}{dT_1} = -\mu a_1 + \frac{b_{12}}{8\Omega} a_1 a_2^2 \sin 2(\theta_2 - \theta_1) - \frac{f}{2\Omega}\cos\theta_1 \\ &a_1 \frac{d\theta_1}{dT_1} = \delta a_1 - \frac{3b_{11}}{8\Omega} a_1^3 - \frac{b_{12}}{8\Omega} a_1 a_2^2 [2 + \cos 2(\theta_2 - \theta_1)] + \frac{f}{2\Omega}\sin\theta_1 \\ &\frac{da_2}{dT_1} = -\mu a_2 - \frac{b_{12}}{8\Omega} a_1^2 a_2 \sin 2(\theta_2 - \theta_1) \\ &a_2 \frac{d\theta_2}{dT_1} = (\delta - \sigma) a_2 - \frac{3b_{22}}{8\Omega} a_2^3 - \frac{b_{12}}{8\Omega} a_1^2 a_2 [2 + \cos 2(\theta_2 - \theta_1)] \end{aligned} \qquad (2)$$

For s*tationary (steady-state)* modes of oscillation ($a_k = const$, $\theta_k = const$, $k = 1,2$) the right hand sides of set (2) are equal to *0*. Omitting the apparent case of an *uncoupled* mode $a_1 \neq 0$, $a_2 = 0$ we focus our attention on *coupled* stationary modes $a_1 \neq 0$, $a_2 \neq 0$ (CSM) which can be considered as a superposition of the linear (synchronized) oscillations in each degree of freedom. By excluding θ_1 and θ_2 from the set (2) a set of two algebraic equations for amplitudes a_k of the CSM's is obtained.

We consider separately cases of undamped ($\mu = 0$) and damped systems. In *undamped* systems phases θ_k satisfy the conditions $\gamma \equiv \theta_2 - \theta_1 = k\pi / 2$ (*k*=1, 2,...), $\theta_1 = \pm\pi / 2$. So, similarly to the case of free oscillations, two types of CSM's exist in undamped systems: a) normal modes (*NM*), for which $\gamma = 0$ or $\gamma = \pi$ (both modes u_k reach their extreme values at the same instant of time); b) elliptic modes (*EM*), for which $\gamma = \pi / 2$ or $\gamma = -\pi / 2$ (extreme values of one mode coincide with zero values of another one).

It is shown that i*n damped systems the CSM's are not exact normal or elliptic modes but they asymptotically approach NM or EM when the energy of oscillations increases (if they exist at large amplitudes).*

The qualitative and quantitative analysis of frequency response curves (for a given excitation force parameter) and amplitudes of both modes as functions of the force parameter was carried out with using known results for free oscillations modes in the considered systems [3, 4]. It has been established earlier, in particular, that in autonomous systems only one normal modes path and one elliptic modes path can exist in 3D space (a_1, a_2, ω). Configuration of these paths (which may be finite or infinite), their number (0 or 1) and stability depend only on the nonlinear coefficients b_{ij}; their origin (if the CSM's paths are infinite) or boundary points (if the CSM's paths are finite) lie on the uncoupled modes paths, and in these bifurcation points the uncoupled modes change their stability.

These features are shown to be peculiar, with certain modifications, also to forced oscillations in the considered systems. There are exist up to two CSM's paths in 3D space (a_1, a_2, f) (for a given frequency of excitation), which are finite or infinite curves depending on the nonlinear coefficients b_{ij}. Their boundary points (one or two bifurcational points) lie on the uncoupled modes path for the excited mode or on the "backbone curve" for the companion mode (the latter case takes place in undamped systems). So the CSM's paths either connect these "uncoupled curves" or go from one of these curves to infinity, similarly to the case of autonomous systems.

In 3D space (a_1, a_2, δ) (δ is used as an excitation frequency parameter, the amplitude of excitation is given) the CSM's paths in damped systems are spatial curves which boundary points lie on the uncoupled modes path. The portion of the latter path between the two points is always unstable. In undamped systems the CSM's path undergoes fission on two infinite branches. Asymptotes of these branches are the backbone curves for the companion mode and for the CSM's (at free oscillations).

In *undamped* systems at *exact* internal and external resonances the CSM's paths in 3D space (a_1, a_2, f) originate from the zero point, i.e. they exist at any small energy; in damped systems the CSM's paths appear after exceeding a certain energy threshold. In *damped* systems the CSM's do not exist at sufficiently small energy of oscillations; they can appear only at amplitude value of the excited mode satisfying condition $a_1^2 > 8\,\Omega\,\mu / |b_{12}|$. The CSM's exist at any large energy of oscillations ($a_1 \to \infty$) if $b_{12}\, b_{22} < 0$.

Numerous results of numerical solution of the algebraic set of equations for a_k (*k*=1,2) in the form of frequency response curves and amplitudes a_k via the excitation force parameter are presented to illustrate peculiarities of steady-state motions at forced oscillations. Types of bifurcational points, their number and stability have been studied for undamped and damped systems.

References

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