SENSITIVITY ANALYSIS AND OPTIMAL DESIGN OF GEOMETRICALLY NONLINEAR 3D FRAMES WITH ACCOUNT FOR STABLE POSTBUCKLING BEHAVIOUR

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Summary The postbuckling behaviour of optimal geometrically nonlinear 3D frames is usually not analyzed since the critical load constraints are included in the optimization problem,[5]. Therefore it is not clear whether behaviour of the optimal frame after buckling is stable or unstable. In order to overcome that it is possible to implement postbuckling constraints into the formulation of optimization problem that take care of the form of nonlinear equilibrium path and modify the design in order to obtain stable behaviour after buckling. To guarantee the stable postbuckling behaviour of the optimal frame we adopt the expression for the change of total potential energy which will allow us to investigate the stability of singular points. Implementation of postbuckling constraints into the problem of determination of the optimal joint positions and cross-sectional parameters of geometrically nonlinear space frames results in minimization of the global mass of the frame subject to elimination of snap-through. The sensitivity analysis of the small change of total potential energy is performed through analytic differentiation with respect to design parameters. Examples of optimal design of space frames are presented.

PROBLEM FORMULATION

Consider an elastic frame structure with large displacements and rotations. The use of corotational approach eliminates the rigid frame rotations prior to strain computation allowing the application of the standard small strain-displacement relationship. The external loads $\Lambda\{P_0\}$ acting on the frame are assumed to be conservative, where Λ denotes the proportional loading parameter. The design vector $\{X\}$ is composed of cross-sectional parameters $\{X_1\}$ and configuration parameters $\{X_2\}$ specifying the positions of member joints. These two classes of parameters are linked to types and groups in order to satisfy constructive requirements.

The objective function is identified with the global mass of the frame which should be minimized, thus

$$W\left(\{X^*\}\right) = \min_{\{X\}\in F} W\left(\{X\}\right) = \sum_k \rho_k a_k\left(\{X_1^*\}\right)\sum_{e\in k} L_e\left(\{X_2^*\}\right), \tag{1}$$

where ρ_e, a_e and L_e denote the density, the area and the length of *e*-th beam member, *k* denotes the type of beams. The feasible domain *F* is specified by the following constraints: constraint on the small change in total potential energy to eliminate snap-through and side constraints on the design variables. The constraint for elimination of snap-through is not considered if the critical load is greater than reference load, Fig.1.

The optimization problem can be formulated as follows:

$$F = \{ \{X\} \mid \quad \Delta\Pi\left(\{p\},\Lambda,\{X\}\right)\big|_{at\,critical\,point} \geq 0, \quad \{X\}_{min} \leq \{X\} \leq \{X\}_{max} \ \}, \tag{2}$$

where

$$\Delta\Pi = \frac{B_1}{6}\Delta\bar{s}^3 + \frac{B_5}{24}\Delta\bar{s}^4 \tag{3}$$

$$B_1 = \{z_1\}^T\left\{\frac{\partial^2\{g\}}{\partial p_i \partial p_j} z_1(i) z_1(j)\right\}, \tag{4}$$

$$B_5 = \{z_1\}^T\left\{\frac{\partial^3\{g\}}{\partial p_i \partial p_j \partial p_k} z_1(i) z_1(j) z_1(k)\right\} - \sum_{r=2}^{n}\frac{3}{\lambda_r}\left(\{z_1\}^T\left\{\frac{\partial^2\{g\}}{\partial p_i \partial p_j} z_1(i) z_r(j)\right\}\right)^2, \tag{5}$$

$\Delta\Pi$ – the change in total potential energy, B_1, B_5 – the stability coefficients, $\{z_1\}$ – the first eigenmode which represents the buckling mode of the frame, λ_r – the eigenvalues of the tangent stiffness matrix $[K_t]$,

$\{g(\{p\},\Lambda,\{X\})\}-$ the equilibrium equations, $\{p\}-$ the displacement vector, $p_i -$ the i-th component of $\{p\}$, $z_1(i)-$ the i-th component of $\{z_1\}$, $\{X\}_{\min}$, $\{X\}_{\max}$ $-$ given lower and upper bounds for the design variables. $\Delta\bar{s}$ has been calculated by Crisfield [6].

PROBLEM APPROXIMATION

One of the ways to effectively solve the large dimension optimization problem (1-2) is in replacing the exact formulation with an approximate one:

$$W(\{X^*\}) = \min_{\{X\}\in\tilde{F}} W(\{X\}) \tag{6}$$

The approximate feasible domain $\tilde{F}$ is specified by the approximation of the postbuckling constraint:

$$\Delta\Pi(\{p\},\Lambda,\{X\})\big|_{\det[K_t]=0} + \nabla^T \Delta\Pi(\{p\},\Lambda,\{X\})\big|_{\det[K_t]=0} \{\Delta X\} \ge 0 \tag{7}$$

$$\{\Delta X\}_{\min} \le \{\Delta X\} \le \{\Delta X\}_{\max}, \tag{8}$$

Calculation of derivatives of $\Delta\Pi$ with respect to design variables $\{X\}$ is referred to the sensitivity analysis, [1-4].

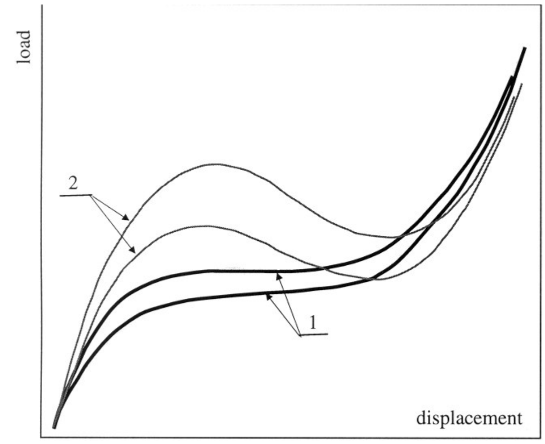

Fig.1. Feasible designs of frame subject to elimination of snap-through.
1-stable load-deflection curves for feasible designs,
2- load-deflection curves for unfeasible designs

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