

**OPTIMAL LAYOUT OF TWO MATERIALS
WITHIN THE CORE LAYER OF A SANDWICH PLATE.
RELAXED FORMULATION AND ITS COMPUTATIONAL ALGORITHM**

Sławomir Czarnecki, Michał Kurşa, Tomasz Lewiński

Warsaw University of Technology, Al. Armii Ludowej 16, PL 00-637 Warszawa, Poland

Summary The paper deals with the minimum compliance problem of a sandwich plate with soft core. The aim is to find an optimal layout of two isotropic materials within the core layer. In the numerical treatment of the relaxed formulation the core layer is modelled by a 2nd rank laminated composite.

Deformation of a sandwich plate of a soft core, see Fig.1, transversely loaded, will be modelled by the Reissner model of 1947, see Sec. 6.3 of Ref.[5]. The bending stiffnesses ($D^{\alpha\beta\lambda\mu}$), $\alpha, \beta, \lambda, \mu = 1, 2$, depend on the properties of the faces and on the distance $2c$. The aim is to optimize the core the transverse shear moduli of which determine the effective stiffnesses ($H^{\alpha\beta}$). The moments ($M^{\alpha\beta}$) and the transverse forces (Q^α) are interrelated with the deformation measures by

$$M^{\alpha\beta} = D^{\alpha\beta\lambda\mu} \varepsilon_{\lambda\mu}(\boldsymbol{\varphi}), \quad Q^\alpha = H^{\alpha\beta} \gamma_\beta(w, \boldsymbol{\varphi}) \quad (1)$$

where $\boldsymbol{\varphi} = (\varphi_1, \varphi_2)$ is a vector of the angles of rotation of the transverse cross sections and w represents the plate deflection; the deformations $\varepsilon_{\lambda\mu}(\boldsymbol{\varphi})$ are defined as symmetric part of the gradient of $\boldsymbol{\varphi}$ and $\gamma_\beta(w, \boldsymbol{\varphi}) = w_{,\beta} + \varphi_\beta$. Here comma means differentiation with respect to x_1 or x_2 .

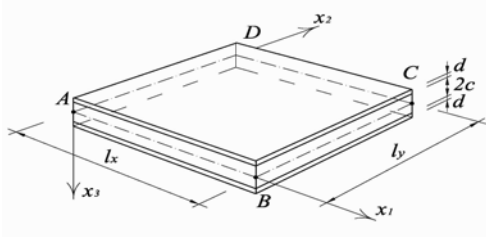


Fig.1 Sandwich plate

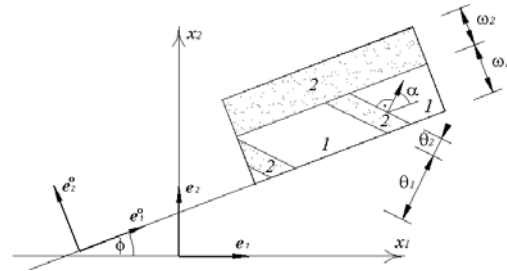


Fig.2 Microstructure of the core layer: a 2nd rank laminate

The variational equilibrium equation of the plate has the form

$$\int_{\Omega} [M^{\alpha\beta} \varepsilon_{\alpha\beta}(\overline{\boldsymbol{\varphi}}) + Q^\alpha \gamma_\alpha(\overline{w}, \overline{\boldsymbol{\varphi}})] dx = f(\overline{w}) \quad \forall \overline{w} \in V \quad f(\overline{w}) = \int_{\Omega} q(x) \overline{w}(x) dx \quad (2)$$

where q represents the transverse loading, Ω is a plate middle plane and V represents the space of kinematically admissible displacement fields. Assume that the core is formed transversely homogeneously of two isotropic materials labelled by $\gamma = 1, 2$ of transverse shear moduli $C_{\gamma}^{\alpha\beta\lambda\mu} = \delta^{\alpha\beta} \mu_\gamma$. Assume that these materials occupy the domains Ω_γ within Ω ; the characteristic functions of the domains being denoted by χ_γ . Let the areas $|\Omega_\gamma| = C_\gamma$ be prescribed. The problem of minimizing the compliance of the plate is put in the form

$$\min \{ f(w) \mid w \text{ solves (1), (2) and } |\Omega_2| = C_2 \} \quad (3)$$

The relaxation of the above problem means admitting the characteristic functions χ_γ to tend to their weak limits $m_\gamma \in L^\infty(\Omega; [0, 1])$, see Díaz et al.[4] and Lewiński and Telega [5]. Let $G_{m_2}^{per}$ be a set of tensors \mathbf{H}^* determining the composite material of the core, of two-phase periodic structure, according to the formulae of the homogenization theory, under the assumption of the area fraction of the material 2 being given as m_2 . Let us define the effective potential for the transverse forces

$$W_s(\boldsymbol{\gamma}, m_2) = \max \{ \boldsymbol{\gamma} : (\mathbf{H}\boldsymbol{\gamma}) \mid \mathbf{H} \in G_{m_2}^{per} \} \quad (4)$$

Thus the effective constitutive relationships read

$$\mathbf{M} = \mathbf{D}\boldsymbol{\varepsilon}(\boldsymbol{\varphi}), \quad \mathbf{Q} = \frac{\partial W_s(\boldsymbol{\gamma}, m_2)}{\partial \boldsymbol{\gamma}} \quad (5)$$

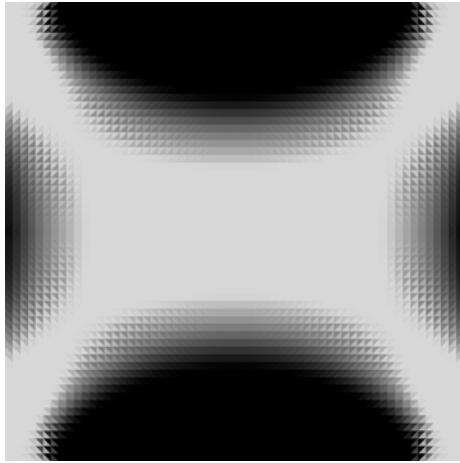
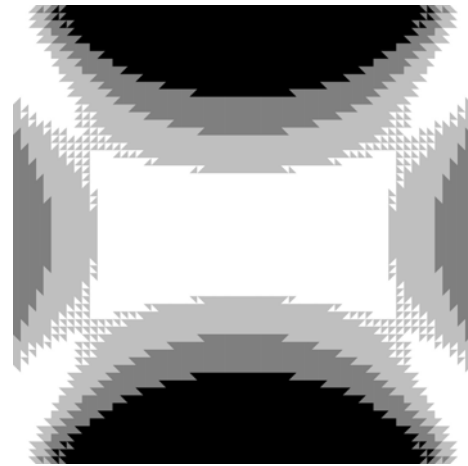


Fig. 3 Optimal distribution of the area fraction m_2 

Fig. 4 Types of microstructures:

1st rank laminate: ; 2nd rank laminate: 
 stronger isotropic material is black;
 weaker isotropic material is printed white

The relaxed problem assumes the form

$$\min \left\{ f(w) \mid w \text{ solves (2),(4),(5), } m_2 \in L^\infty(\Omega; [0,1]), \int_{\Omega} m_2 dx = C_2 \right\} \quad (6)$$

In the numerical computations the set $G_{m_2}^{per}$ has been replaced by the set comprising the 2nd rank laminates, see Fig.2, characterized by the design variables $\omega_2, \theta_2, \alpha, \phi$. The effective moduli (H^{ab}) are determined by the formula of Tartar, see Sec. 5.6.4 in Lewiński and Telega [5], applied iteratively. The equilibrium problem is solved with using the DSG finite element, see Bletzinger et al.[2], free of shear locking and satisfying the convergence consistency criteria of Strang. The optimization has been performed with using the COC method and the updating schemes reported in Bendsøe [1]. A similar algorithm has been previously applied to thin plates in Czarnecki and Lewiński [3]. The optimal inclination of ribs is given by the formula

$$\hat{\phi} = \chi - \chi_0, \quad \chi_0 = \frac{1}{2} \arctan \left(\frac{2H_0^{12}}{H_0^{11} - H_0^{22}} \right) \quad (7)$$

where $\chi = \angle(\mathbf{e}_1, \boldsymbol{\gamma})$ and (H_0^{ab}) refer to the basis ($\mathbf{e}_1^0, \mathbf{e}_2^0$), see Fig.2. Thus we find the optimal distributions of m_2 in the core layer, see Fig.3, where the assumption $\alpha = 0$ is used. The optimal layout is composed of the weaker material, of the stronger material and of laminates of 1st and 2nd rank. The layouts in Figs.3,4 refer to the quadratic plate ($l_x = l_y = 1.0$ m, $c=0.01$ m, $d=0.002$ m) clamped along the sides AB, CD and supported along BC, DA such that there $w=0, \varphi_2=0$. The plate is subjected to a uniform transverse loading. The faces are made of an isotropic material of moduli $E=130.0$ GPa, $\nu=0.3$; the core is made of two materials of moduli $E_1=4.835$ GPa, $\nu_1=0.3$; $E_2=13.0$ GPa, $\nu_2=0.3$. The stiffnesses of the sandwich plate are computed according to the formulae of Sec.6.3 of Ref.[5]. The stronger material occupies 0.51 part of the domain Ω . The computations have been performed with using the updating scheme, see [1], with the damping factor $\eta=0.75$ and the move limit $\zeta=0.013$. The optimal layouts of the area fractions and the underlying microstructures compare favorably with 3D layouts found within the relevant relaxed formulation by homogenization.

References

- [1] M. Bendsøe.: Optimization of Structural Topology, Shape, and Material. Springer. New York 1995.
- [2] K.U. Bletzinger, M. Bischoff, E. Ramm.: A Unified Approach for Shear-Locking-Free Triangular and Rectangular Shell Finite Elements. *Computers and Structures* **75**: 321–334, 2000.
- [3] S. Czarnecki, T. Lewiński.: Optimal Layouts of a Two-Phase Isotropic Material in Thin Elastic Plates. European Conference on Computational Mechanics, June 26-29, Cracow, Poland, 2001.
- [4] Díaz A.R, Lipton R, Soto C.A.: A new formulation of the problem of optimum reinforcement of Reissner-Mindlin plates. *Comp.Meth.Appl.Mech.Engrg.* **123**: 121-139, 1995.
- [5] T. Lewiński, J.J. Telega.: Plates, Laminates and Shells. Asymptotic Analysis and Homogenization. World Scientific. Singapore, New Jersey, London, Hong Kong, 2000.