ON THE STABILITY OF THE SKY-HOOK

Alois Steindl and Hans Troger
Institute of Mechanics, Vienna University of Technology,
Wiedner Hauptstrasse 8-10, A-1040 Vienna, Austria

Summary The sky-hook, that is a string forming a connection from the surface of the Earth to a satellite in geostationary orbit, which may be used as track for an Earth to space elevator, is an old dream of mankind, originating about 100 years ago in Russia. Besides the question of feasibility from a technological point of view also the question concerning the stability of such a configuration has not yet been completely solved. Under the assumption that a proper material (carbon nanotubes) is available making the connection possible from the technological point of view, we address the question of stability of the radial relative equilibrium of a very long tapered string, which rotates synchronously with the Earth and reaches from the surface of the Earth up into the sky. The solution of the stability problem for different string materials is given by application of the Reduced Energy Momentum Method. Since the string is orbitally unstable, we connect it to a satellite in geostationary orbit and give the minimum mass of the satellite to stabilize the whole system.

MOTIVATION AND HISTORY OF THE PROBLEM

One of the main problems of modern space exploration and space technology is the high cost of sending a payload from the surface of the Earth into space. Depending on the destination in space, in the year 2000, these costs are about $10^4 - 10^6$ US Dollars for one kilogram of payload.

Hence for a long time there have been other ideas around for a cheaper way of transporting payloads into space orbit. One of the most attractive was proposed in 1960 by the Russian scientist Yu. Artsutanov ([1]) namely to build a celestial elevator from the surface of the Earth to a satellite in geostationary orbit. In order to compensate for the weight of the string, the string must be extended beyond the geostationary radius (approximately 35800 km altitude). The part of the cable extending beyond the geostationary orbit must have a length which is several times the length to the geostationary orbit ([3]) if it is designed for minimum weight (see also Fig. 2).

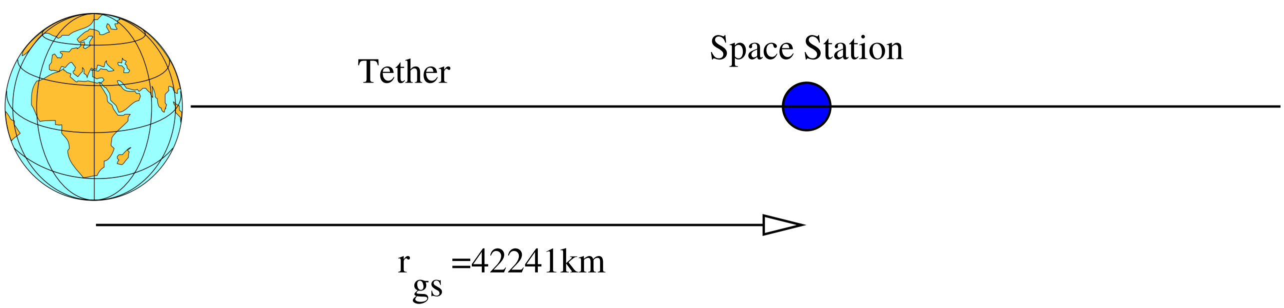

Figure 1. Cable connecting a satellite in geostationary orbit with the surface of the Earth. As counterweight another cable must be deployed outside the geostationary hight. For a minimum weight design its length and its shape are given in Fig. 2

This idea makes already use of the fact that a massive string moving on a circular orbit around the Earth, under the action of gravitational and centrifugal forces, finally will reach a relative equilibrium position, which is its radial position. It can be shown that in this equilibrium position the string is under tension ([2]). In order to obtain a minimum weight design the shape of the cable must be tapered such that it is thickest at the point of highest tension which is at the geosynchronous radius and thinnest where the tension is lowest (at its ends) (Fig. 2). Such a cable configuration could be used as track for a space elevator to provide easy transportation of payloads to the geosynchronous orbit and beyond ([7]).

REALISATION AND STABILITY ANALYSIS

Until 1991 these ideas were purely academic, since no material was available to realize such a project. However, at that time socalled "carbon nanotubes" ([6]) were discovered, which are cylindrical macromolecules composed of carbon atoms which are formed from a flat periodical hexagonal latice with the thickness of the size of an atom. Single walled nanotubes have been produced with a diameter of a few Nanometers ($1[nm] = 10^{-9}[m]$) and a length of the order of Centimeters. Hence sofar an aspect ratio of order 10^7 has been reached. Single walled nanotubes form the building block of multi-walled nanotubes. Moreover, it is conceived to have bundels of nanotubes. Forming nanoropes from nanotubes a theoretical strength of 100 times higher than steel can be expected but with only one-sixth of the weight of steel. Moreover, besides their extreme strength they also allow large strains, up to 16-24 %.

The ratio between tensile strength and density is crucial for the taper ratio of the string, that is, the ratio between the cross section at the geosynchronous orbit to the cross sectional area of the cable at Earth. For example from the calculations performed in [3] the taper ratio required for steel would be 1.7×10^{33}, for Kevlar 2.6×10^8 and for carbon nanotubes only 1.5.

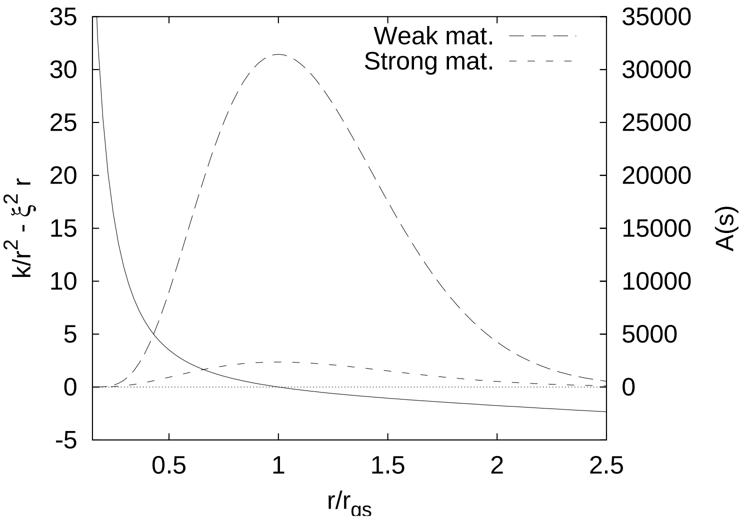

Figure 2. For two cables in the radial relative equilibrium configuration on the geostationary orbit the areas of their cross-sections for carbon nanotubes with different admissible tensile stresses σ_c are given. The curve denoted by ”strong mat.” corresponds to maximum strength nanotubes. The solid curve gives the force acting on a cable element.

A careful, practically relevant investigation of stability of the the system’s relative equilibrium would have to take into account various perturbations such as the gravitational attraction of the Moon, atmospheric drag and moving payloads up and down the cable. However, even the simpler question of stability of a long unperturbed tapered string in the spherical symmetric Newtonian gravitational field is still an open problem, because in [2] and [4] it is shown that a dumbell satellite, which is a system of two point masses connected by a massless rigid rod, possesses a stable radial relative equilibrium position on a circular orbit around the Earth only if the distance between the two masses is not too large. Here by large we understand a distance of the order of the radius of the orbit.

Hence the purpose of this paper is to investigate whether a continuous massive cable, necessary for the realization of the skyhook, has a stable radial relative equilibrium. Since it will turn out that the radial relative equilibrium position of the tapered cable is unstable we include at the geostationary height a satellite and ask the question, first, whether it has a stabilizing effect and since this is the case we give, second, the answer to the question what is its minimum necessary mass. The proper theory to answer these questions is the Reduced Energy Momentum Method ([4], [8], [5]).

In addition to explain some surprising results of the stability investigation of the continuous system we also study the simpler problem of a string pendulum in orbit around the Earth which qualitatively yields the same results.

CONCLUSIONS

Our main result is that the radial relative equilibrium position of a massive tapered string moving in geostationary orbit around the Earth, made of the new material called “carbon nano tubes”, reaching from the surface of the Earth beyond the geostationary radius, is orbitally unstable. However, if the string is connected to a massive satellite moving in geostationary height the whole system can be stabilized. We give the minimum value of mass of the satellite to have this stabilizing effect on the whole arrangement.

ACKNOWLEDGEMENT

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