MODELLING OF COUPLING BETWEEN INDUCED ANISOTROPIC DAMAGE AND PERMEABILITY IN ROCKS

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Summary The damage state is represented by a second order tensor. The evolution of damage is determined from a crack propagation criterion. The free enthalpy function of cracked solid is expressed by using micromechanical considerations. It is assumed that cracks exhibit normal opening which is coupled with the crack propagation. The overall permeability of the RVE is obtained by a space integration method taking into account crack opening in each orientation.

FORMULATION OF THE MODEL AND SIMULATIONS

In this paper, it is assumed that rocks are submitted to compression-dominated stresses. The interaction between cracks can be neglected before coalescence of microcracks. The initial behaviour of materials is isotropic and the anisotropy is fully induced by preferential distribution of microcracks.

Damage variable

A second rank tensor is used for the distribution approximation of crack density and orientation. The crack density in the orientation defined by the unit vector $\vec{n}$ is defined as $\rho(\vec{n}) = l^3(\vec{n})/\Omega$, with l being the average radius of penny shaped microcracks and Ω the volume of the representative volume element (RVE). As most rocks contain an initial state of microcracks due to formation process, the induced damage is defined as the relative evolution of microcracks by external loading. For the sake of simplicity, an isotropic distribution is assumed for the initial state of microcracks. The crack density is function then defined as $\rho(\vec{n}) = (l^3(\vec{n}) - l_0^3)/\Omega$ where l_0 is the initial value of crack radius. The macroscopic damage tensor is obtained by orientation integration of density function over the surface of unit sphere:

$$\underline{\underline{D}} = \frac{1}{4\pi}\int_{S^2} \frac{l^3 - l_0^3}{b^3}\vec{n}\otimes\vec{n}ds = \frac{1}{4\pi}\int_{S^2}(r^3 - r_0^3)\vec{n}\otimes\vec{n}ds \tag{1}$$

The variable b is a critical length for crack coalescence leading to instable crack propagation and macroscopic failure.

Constitutive equations of cracked materials

According to micromechanical analysis (Kachanov 1993, Pensee et al. 2002) and by using a second rank tensor approximation of crack distribution, the free enthalpy function of cracked materials can be expressed with the invariants of Cauchy stress tensor σ and damage tensor.

$$G(\sigma,\underline{\underline{D}}) = \frac{1+\nu_0}{2E_0}tr(\sigma.\sigma) - \frac{\nu_0}{2E_0}(tr\sigma)^2 + a_1 tr(\underline{\underline{D}})(tr\sigma)^2 + a_2\, tr(\sigma.\sigma.\underline{\underline{D}}) + a_3\, tr(\sigma)tr(\sigma.\underline{\underline{D}}) + a_4\, tr(\underline{\underline{D}})tr(\sigma.\sigma) \tag{2}$$

where E_0 and ν_0 is respectively the elastic modulus and Poisson's ratio of undamaged matrix. The four parameters a_i, depending on the elastic properties of undamaged matrix, are used to describe degradation of elastic properties of cracked materials. The constitutive equations are obtained from standard derivation of (2):

$$\varepsilon - \varepsilon^r(\underline{\underline{D}}) = \frac{\partial G(\sigma,\underline{\underline{D}})}{\partial\sigma} = \frac{1+\nu_0}{E_0}\sigma - \frac{\nu_0}{E_0}(tr\sigma)\delta + a_2(\sigma.\underline{\underline{D}} + \underline{\underline{D}}.\sigma) + a_3\left[tr(\sigma.\underline{\underline{D}})\delta + (tr\sigma)\underline{\underline{D}}\right] + 2a_4(tr\underline{\underline{D}})\sigma \tag{3}$$

where $\varepsilon^r(\underline{\underline{D}})$ is the irreversible strain tensor due to residual opening of microcracks after complete unloading of materials (Shao and Rudnicki 2000).

Damage evolution and crack propagation

In the framework of thermodynamics, the damage evolution law is determined by definition of a dissipation potential as a function of the conjugated forces associated with the damage tensor. In this work, a simpler approach is preferred in order to facilitate the determination of model parameters. The damage evolution law is directly linked to a crack propagation criterion based on the fracture mechanics. Based on experimental data from triaxial compression tests on rocks, the crack propagation is controlled by both the mean stress and deviatoric stress. The following criterion is proposed to describe crack propagation in the orientation $\vec{n}$:

$$F(\sigma,r,\vec{n}) = \sqrt{r}\left[\sigma_n + f(r)q(\vec{n})^2\right] - c_r \le 0\,,\ \sigma_n = \sigma_{ij}\, n_i\, n_j\,,\ q(\vec{n}) = H(\sigma^d_{ij}\, n_i\, n_j)\,,\ \sigma^d = \sigma - (\sigma_{kk}/3)\delta \qquad (4)$$

where σ_n and $q(\vec{n})$ is respectively the normal stress and deviatoric stress applied on crack. $f(r)$ is a hardening function controlling kinetics of crack propagation.

Variation of permeability

The variation of permeability is due to crack aperture. In this work, we consider the case where rocks are submitted to compression-dominated stresses. The crack aperture is related to crack propagation due to crack surface asperity. Further, it assumed that the undamaged material has an initial porosity and all induced cracks are connected to the initial porosity. The Darcy law is assumed valid for fluid flow through cracks and through the RVE. The total permeability is the sum of the initial permeability k^0_{ij} and crack permeability k^c_{ij}. At this stage, we assume a linear relationship between crack aperture and radius such as $e = (l - l_0)/\chi$. Based on the works of Oda (1985), a set of three random variables $(\vec{n},\tilde{l},e)$ is defined for crack orientation, relative radius ($\tilde{l} = (l - l_0)$ and aperture. The unit vectors normal to cracks are oriented inside a small solid angle dS around $\vec{n}$, and the radius and the apertures range from $\tilde{l}$ to $\tilde{l} + d\tilde{l}$ and from e to $e + de$ respectively. A probability density function $E(\vec{n},\tilde{l},e)$ is introduced in such a way that $2E(\vec{n},\tilde{l},e)dSd\tilde{l}\,de$ gives the probability of $(\vec{n},\tilde{l},e)$. After integration over the entire solid angle on the unit sphere, the crack permeability tensor can be expressed as a function of a crack tensor:

$$k_{ij} = \lambda(p_{kk}\delta_{ij} - p_{ij})\,,\ p_{ij} = \pi\hat{\rho}\int_0^\infty \int_0^\infty \int_{S^2} \tilde{l}^2 e^3 n_i n_j E(\vec{n},\tilde{l},e)dSd\tilde{l}\,de \qquad (5)$$

where $\hat{\rho} = \hat{m}/\Omega$ is the volume (number) density of cracks. The relative variation of crack length $\tilde{l} = rb$ is determined by the crack propagation criterion (4).

The proposed model is applied to a sandstone. In Figure 1, we present the simulation of a triaxial compression test. The variation of the axial permeability as a function of axial strain is presented on Figure 2 for different confining pressures. Such results are qualitatively in agreement with experimental data obtained in brittle rock materials.

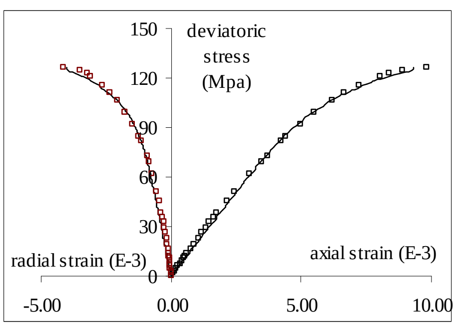

Figure 1: Simulation of a triaxial compression test

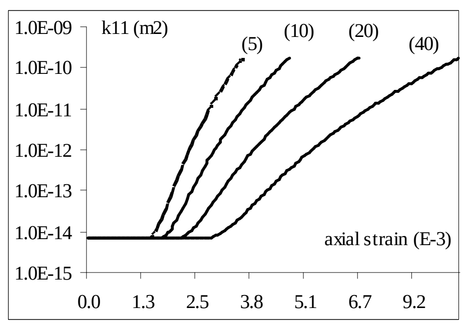

Figure 2: Variation of permeability due to crack propagation in triaxial compression tests with different confining pressures

CONCLUSIONS

An anisotropic damage model is proposed by taking into account the variation of permeability due to crack aperture. The simulations by the model are qualitatively in agreement with experimental observations. However, further validation is necessary to check the validity of the model.

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