# ON OPTIMAL CONTROL OF A QUASIVARIATIONAL INEQUALITY ARISING FROM A VISCOELASTIC CONTACT PROBLEM

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**Extended Abstract**:
**Introduction**: In this article, we consider an unilateral frictional contact between a viscoelastic body and a rigid foundation. A weak formulation of the recast problem yields a parabolic quasivariational inequality corresponding to a reflected process. We consider the minimization of

$$J(\psi) = \int_0^\tau \int_\Omega \left| \dot{\phi}_d - \dot{\phi} \right|^2 dxdt + \int_{\Gamma_C} \left| \phi_d - \phi \right|^2 ds + r \left\| \psi \right\|^2 \tag{0.1}$$

subject to the quasivariational inequality. Similar work was done by Capatina [4] in the case of elastic contact involving an elliptic quasivariational inequality and Amassad et.al. [1] in the case of quasistatic frictional contact involving an elliptic variational inequality. In this work, however, the state of our system is governed by a parabolic quasivariational inequality and we seek to minimize a pertinent cost functional. This is the thrust of our present work.
In (0.1) above the dots represent time derivatives, $r > 0$, $\phi = \phi(u, t)$ is the transform variable, $\phi_d$ is a target displacement and $\psi$ is the distributed control.
**Problem setting**: The physical setting is the following. A viscoelastic body occupies a bounded domain $\Omega \subset \Re^2$ with a regular boundary partitioned into 2 mutually disjoint measurable portions $\Gamma_D, \Gamma_C$, such that meas $\Gamma_X > 0$ ($X = D, C$). We denote by $\vec{n}$ the unit outward normal. The body is clamped on $\Gamma_D$, a volume force is applied in $\Omega$. $\Gamma_C$ is such that the body may come into frictional contact with the rigid foundation.
The mechanical-mathematical setting is as follows:

**Problem $P_1$**
Find the displacement field $\vec{u} : \Omega \rightarrow \Re^2$ such that

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$$\lambda\vec{u}_{xxt} + \mu\vec{u}_{xx} + f(\vec{u},t) = \vec{u}_{tt} \text{ in } \Omega \subset \Re^2 \; ; \tag{0.2}$$

$$\vec{u} = 0 \text{ on } \Gamma_D \text{ and } ; \tag{0.3}$$

$$\sigma_n(u) = 0 \text{ and } u_n < g \text{ on } \Gamma_C \tag{0.4}$$

$$u_n = g \text{ on } \Gamma_C, \text{then } \sigma_n(u) < 0 \tag{0.5}$$

$$\text{if } |\sigma_T(u)| < \nu_F|\sigma_n(u)| \text{ then } u_T = 0 \tag{0.6}$$

$$\text{if } |\sigma_T(u)| = \nu_F|\sigma_n(u)| \text{ then } \exists\, \kappa \geq 0 \text{ such that } u_T = -\kappa\sigma_T(u) \tag{0.7}$$

$$\nu_F \in L^\infty(\Gamma_C), \;\; \nu_F \geq \nu_o > 0 \text{ a.e. on } \Gamma_C. \tag{0.8}$$

Above $\sigma = \mu\vec{u}_x + \lambda\vec{u}_{xt}$ is the stress tensor for the material, $\vec{u}$ the displacement vector, (0.6) to (0.8) represent the Coulomb friction assumed on $\Gamma_C$ and subscripts $n$ and $T$ respectively denote normal and tangential components.

$P_1$ is a classical formulation of the problem . Under a transformation, it is expressible as:

**Problem** $P_2$

Find the displacement field $\vec{u} : \Omega \to \Re^2$ such that

$$-\lambda\vec{\phi}_{xx} + \mu\vec{\phi}_t + f(\vec{\phi},t) = 0 \text{ in } \Omega \subset \Re^2; \tag{0.9}$$

$$\vec{\phi} = 0 \text{ on } \Gamma_D \text{ and} \tag{0.10}$$

$$\phi_n - h_n(u) \leq 0 \text{ on } \Gamma_C \tag{0.11}$$

Following [4], $P_2$ can be recast in its weak form (in an appropriate space) as a parabolic quasivariational inequality. In this paper, we establish existence of solution to the quasivariational inequality. We next give statements for the optimal control of the quasivariational inequality subject to (0.1). In addition, the case of an elastic composite foundation is also considered using an appropriate energy function in large deformation.

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