

## FRICTIONAL SLIDING OF A MULTISLIP SYSTEM

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*Summary* The brittle/ductile transition in rocks has an essential influence in determining the strain rate in lithospheric plates. With this motivation we attempt to study the micromechanics of an elastic medium with interfaces whose slip is governed by rate and state friction. As a preliminary analysis we study the linear stability of the steady-state slip of a finite number  $N$  of parallel interfaces caused by a constant velocity applied at one edge of the medium. We show that interfacial slip can occur with one or two different slip rates if the steady-state friction law displays a minimum as for dry friction. Our results suggest that, when active slip on all  $N$  interfaces is unstable, the medium will select a smaller number of interfaces which continue to slide, the others stopping.

### INTRODUCTION

The mechanical behaviour of the earth's lithosphere is broadly described by the combination of a pressure-dependent brittle behaviour and a thermally activated viscoplastic one, called ductile. The brittle behaviour corresponds to the frictional sliding along preexisting faults and can roughly be modelled by a plastic material with a Drucker-Prager criterion. Concerning the mechanics of Plate tectonics, it has been shown that the brittle behaviour is essential because the brittle/ductile transition determines the strain rate of the lithospheric plate in relation to the forces equilibrium driving the plate [2]. This is why we are attempting to understand the micromechanics of such a brittle medium in order to propose homogenized constitutive models. As a preliminary approach, we study the linear stability of the steady sliding of  $N$  parallel frictional discontinuities in an elastic body.

### THE PROBLEM

We consider a homogeneous elastic medium in mechanical equilibrium of thickness  $H$ , infinite in the horizontal direction  $x$  and divided by  $N$  horizontal interfaces unable to be deformed and along which friction phenomena occur. We consider rate and state friction laws defined by

$$\tau = F(\sigma, \dot{\delta}u, \psi) \quad \text{and} \quad \dot{\psi} = -G(\sigma, \dot{\delta}u, \psi). \quad (1)$$

Such friction laws relate the shear stress  $\tau$  to the compressive normal stress  $\sigma$  and the rate of slip  $\dot{\delta}u$  along the interface. The state of the asperity contacts is modelled by a single state variable  $\psi$ . It is supposed that  $F_V > 0$ ,  $F_\psi > 0$  and  $G_\psi > 0$ . The lower boundary is fixed and the upper one moves with a constant horizontal velocity  $V_H$ . A constant pressure  $P$  is applied on the upper boundary. Assuming the displacement such that  $\mathbf{u} = u(y, t)\mathbf{e}_x$ , it is found that the normal stresses are homogeneous and equal to  $-P$  and that the shear stress depends only on the time:  $\tau(t)$ . The boundary value problem, the stress continuity on each interface and the Hooke's law give a relation between the displacement  $u(y, t)$  in the layer  $i$  and all the displacement jumps  $\delta u_j(t)$  of each interface  $j \leq i \leq N$ . For  $y = H$ , the time derivative of this relation leads to a differential equation for the shear stress  $\dot{\tau}/k + \sum_{i=1}^N \dot{\delta}u_i = V_H$  where  $k = \mu/H$  is a stiffness parameter. Now, regarding the steady-state, the previous equation relates the  $N$  slip rates by

$$\sum_{i=1}^N V_i = V_H. \quad (2)$$

We have denoted  $V_i = \dot{\delta}u_i$  the steady-state slip rate on each interface. By the equation (1)<sub>2</sub> the state of each interface is a function of its corresponding slip rate  $\psi_i^{ss} = \psi_i(V_i; P)$ . In (1)<sub>1</sub>, it leads to  $N - 1$  equations  $\tau = F^{ss}(V_i; P)$  giving with (2) the nonlinear system to solve to find the steady-state slips  $V_i$ . Depending on the properties of the function  $F^{ss}$ , different cases can be envisaged. If  $F^{ss}$  is a monotonic function of  $V_i$  we expect a unique slip rate for the interfaces. If  $F^{ss}$  presents an extremum as observed in the case of dry friction [1], two slip rates  $V_1$  and  $V_2$  are expected. If there is a lubrication or a threshold at  $V_i = 0$ , the interfaces could slip with three different velocities.

### MAIN RESULTS OF THE LINEAR STABILITY ANALYSIS

The stability of a steady-state solution is analysed by usual technique consisting of the linearization of the equations of perturbations around the steady state. It yields an eigenvalue problem defined by a matrix depending on a critical stiffness

$$k_c = F_\psi G_V - F_V G_\psi = -G_\psi \frac{d\tau^{ss}}{dV} \quad (\tau^{ss} = F^{ss}(V_i)), \quad (3)$$

the partial derivatives of  $F$  and  $G$  and parametrized by  $k$  and  $V_H$ . It is denoted  $M(k_c^{(l)}, F_V^{(l)}, F_\psi^{(l)}, G_V^{(l)}, G_\psi^{(l)}; k, V_H)$  where  $l$  denotes the number of distinct  $V_i \neq 0$ . Note that  $k_c > 0$  corresponds to a velocity-weakening steady-state friction law. It has been shown that the velocity-weakening leads to an unstable sliding with stick-slip in the case of a single interface [4, 3]. We show that  $k_c^{(i)}/F_V^{(i)}$  may be an eigenvalue of  $M$  whose algebraic multiplicity depends on the number of the interfaces which slip with the same slip rate. Depending on the stability of the configuration of sliding, *i.e.* one, two or three slip rates, there is a selection of the number of the activated interfaces. We illustrate this result in the case of dry friction in the following.

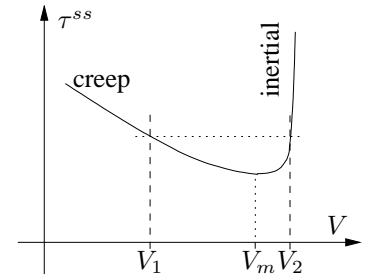
### One steady-state slip rate

When the friction allows only one slip rate for all the interfaces, then  $V_i = V_H/N \forall i$  because of (2). Hence, the determinantal equation is  $(k_c/F_V - s)^{N-1} [s^2 - s(k_c - Nk)/F_V + NkG_\psi/F_V] = 0$ . If  $N > 1$ , we show that if  $k_c < 0$  the sliding is stable for all the interfaces. If  $k_c > 0$ , the biggest eigenvalue is  $k_c/F_V$  and the slip is unstable. It is important to note that, in this configuration, the most unstable mode is real and then the stick-slip instability as in the single-interface problem is not the most unstable mode. Looking at the associated eigenvector, it is remarkable that the presence of many interfaces allows the system to be unstable while the shear stress remains constant. The instability criterion of this configuration is then  $k_c > 0$ . That is, the slip rate  $V_H/N$  has to correspond to a velocity weakening behaviour of the friction law. So, if the friction law shows a minimum for  $V = V_m$ , the sliding becomes unstable at  $N$  fixed if  $V_H$  is sufficiently lower than  $NV_m$ . If  $V_H$  is imposed, the sliding is unstable if  $N$  is too large. Thus, we expect a *critical number of interfaces*  $N_c \simeq V_H/V_m$  such that the sliding is unstable if  $N > N_c$ . About the evolution of an unstable system, the eigenvector associated to  $k_c/F_V$  suggests that  $N - 1$  of the interfaces have to slow down whereas one of them has to speed up. We then expect that an unstable system evolves from a configuration with  $N$  interfaces sliding at  $V_H/N$  towards a system which slips at  $V_H$  just on one interface. Then, depending on the value of  $V_H$  this single-interface sliding can be unstable and present a Hopf bifurcation. A question then arises: because the shear stress has to increase, is this stick-slip instability able to promote a slip on another interface?

### Two steady-state slip rates

We consider a convex steady-state friction law with a minimum at  $V = V_m$  so that two steady-state slip rates can exist: one in the creep regime, the other in the inertial one (cf. Fig. 1). By convention  $V_1 < V_2$ . Then,  $n$  interfaces slip at  $V_1$  and  $m$  at  $V_2$  such that  $n + m = N$  and  $nV_1 + mV_2 = V_H$  by (2). The roots of  $M$  are given by the polynomial  $P_3(s)(s_1 - s)^{n-1}(s_2 - s)^{m-1} = 0$  where  $P_3(s)$  is a cubic and  $s_i = k_c^{(i)}/F_V^{(i)}$ . Thus, 5 eigenvalues are expected.

Note that the partial derivative of  $G$  with respect to  $V$  evaluated at  $V_1$  is such that  $G_V^{(1)} > 0$  because  $k_c^{(1)} > 0$ . We show that the nature of the most unstable modes changes depending on  $n$  and  $G_V^{(2)}$ . When  $n \neq 1$ , the sliding is always unstable because, at least,  $s_1$  is a positive eigenvalue of  $M$ . This mode leaves the shear stress  $\tau$  constant and suggests that  $n - 1$  interfaces stop and one of them speeds up. However, we show that there always exists a critical value  $G_V^* < 0$  of  $G_V^{(2)}$  such that the most unstable mode is a root of  $P_3$  if  $G_V^{(2)} < G_V^* < 0$ . Then, the sliding is unstable on all the interfaces with a growth of  $\tau$ . If this mode is real, it suggests that the  $n$  interfaces at  $V_1$  speed up whereas the  $m$  others slow down and can stop. If the mode is complex, stick-slip phenomenon appears on each interface. When  $n = 1$ ,  $s_1$  is no longer a mode and a stability criterion is obtained by the Routh-Hurwitz criterion which relates the coefficients of  $P_3$ .



**Figure 1.** Dry friction law (cf. [1])

## CONCLUSIONS

With this model problem, we show that a heterogeneous medium holding a large number of dry frictional interfaces is unstable when it is sheared because the effective velocity  $V_H/N$  is low and located in the creep regime. It is suggested that the system then jumps to a single interface sliding in the inertial regime. If  $V_H \sim NV_m$ , the interfaces can slide with two different slip rates. This configuration is unstable and a selection of the number of stable interfaces is expected. But the dynamics of such a selection is unreachable by the present analysis and an experiment would be helpful and very interesting to perform, especially to determine if  $G_V$  can change its sign. Furthermore, the richness of behaviours of this system should give new insights concerning earthquake dynamics and the localisation of deformation in brittle media.

## References

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