THERMOMECHANICS MODELING OF TWO SOLIDS IN CONTACT: APPLICATION TO TOTAL HIP ARTHROPLASTY

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Summary This study includes firstly a theoretical formulation of thermomechanical frictional contact between two defomable solids based on the principles of thermodynamics, and secondly a numerical solution of the resulting non-differentiable equations using an augmented Lagrangian and a generalized Newton method to overcome the contact and friction non-linearities. Finally, the model is applied to the thermomechanical problem of the cement polymerization during a Total Hip Arthroplasty for illustration.

INTRODUCTION

After cemented Total Hip Replacement, failures at stem-cement interface and at bone-cement interface mainly resulted from the occurrence of abnormally high cement stresses and excessive micromotions. Bone-cement slipping leaded to a necrosis of bone that interdigitize with the cement. Mechanical and thermal problems at the interface are very sensitive to changes of contacting boundaries of the bone, cement and implants. This sensitivity is increased by thermoelastic distortion. It may have important consequence of the long-term reliability of the implant mechanical anchorage, particularly on the generation of microcraks within the cement mantle. Some biological complications might be related to the heat generated by the exothermic polymerization of the orthopaedic cement. Bone necrosis at the bone-cement interface may occur during and after the surgical implantation. This may induce bone resorption at the bone-cement interface, leading implant lose. Computational techniques are more and more used to calculate these mechanical and thermal variables trying to simulate clinical situations. However, coupling of all of these variables increases the problem difficulty. To name but several, stem temperature, cement temperature, polymerization temperature, thermoelastic properties of bone, cement and stem including the shrinkage stresses of the cement, certainly affect the quality of the anchorage either immediately after operation or in the long-term. As a starting point, accurate calculation of the bone-implant relative micromotions and interfacial stresses becomes a keypoint for any quantitative analysis of the mechanical environnement acting on the bone tissue end cells. Some numerical algorithms are also very sensitive to solution parameters and it is always reasonable to question about exactness of theoretical models and efficiency of numericalcomputations. In the literature, both theoretical and numerical formulations of thermomechanical contact problems are elaborated in simplified forms less coherent than pure contact mechanics. Contact mechanics is generally treated as a two-body problem but introduces a third body for thermal aspects. In consequence, a formulation more rigorous and coherent is necessary. This fact constitutes the motivation of the present work. A theory of contact thermomechanics between two deformable bodies is proposed together with numerical methods. They are applied to the problem of bone necrosis due to exothermic polymerization of the cement during the cemented total hip replacement.

THEORETICAL FORMULATION AND NUMERICAL RESOLUTION

First, a theoretical formulation of contact thermomechanics is developed in the continuum setting coherent with thermomechanics ofdeformable solids. An interfacial energy equation is elaborated based on the two solids approach withoutthe introduction of the third body. The variable gap of temperature, defined by the difference in temperature between surfaces of contact, and heat flows atthe interface are defined. This formulation took into account the energy equation of each solid constituting the system and the non-additivity of internal energy of the two solids [1]. This non-additivity is due either to the presence of forces of volume and the heat transfers per radiation, or with the localization of a part of the internal energy of each solid constituting the system of contact on the free face of the solids. Second, an entropy inequality at the interface was derived from the inequality of the entropy of the global system in contact and the existence of an entropy production at the level of the interface [2]. It results in thermodynamics restrictions on heat conduction laws through the interface. Third, we propose a interfacial constitutive law of the thermomechanics of contact [3]. This law included a) the thermal unilateral contact, ie the conduction of heat due to the difference in temperature between the two bodies and, b) thermomechanical friction including heat generation. Second, two numerical approaches, based on the penalty and augmented Lagrangian methods, are developed in order to solve unilateral contact thermal problems with and without friction. The penalty method transforms the constrained and non-differentiable thermomechanical contact problems into unconstrained, differentiable but approximated forms. The augmented Lagrangian method treats constrained thermomechanical contact problems and renders exact differentiable and unconstrained formulations. For these two approaches, non-linearities are solved using the generalized Newton method. The numerical algorithm developed in this study was implemented in the finite element numerical program TACT (EPFL, Switzerland).

CLINICAL APPLICATION

Finite element model of the cemented hip stem

A hip stem with a cement mantle of uniform thickness was digitized and numerically inserted in the reconstructed femur following typical surgical procedure. We considered a titanium stem ($Ti6Al4V$) (Young's modulus $E = 110'000MPa$, Poisson's ratio: $\nu = 0.3$) and a chromium-cobalt stem (Young's modulus: $E = 200'000MPa$, Poisson's ratio: $\nu = 0.3$). For each of them, two interfaces (stem-cement and bone-cement) were modelled as unilateral frictional thermocontact. The Young's modulus of the cement was $2200MPa$. Loading conditions corresponded to the single limb stance situation of a gait cycle. The load bearing on the femoral head was simulated with a force of magnitude three times body weight (Patient weight: $600N$) and decomposed into axial, in-plane and out-plane directions. Muscle forces (gluteus minimus, medius, maximus and psoas) were also implemented.

1. The *mechanical analysis* includes three points. First the titalium alloy implant was compared to the chromium-cobalt implant. Cementthickness was set to $4mm$ in thiscase. The friction coefficients at the bone-cement interface and at the stem-cement interface were set to $\mu_{bc} = 1.0$ and $\mu_{sc} = 0.40$ respectively. Secondly, the sensitivity of the results with respect to cement thickness was investigated. The coefficient of friction at the bone-cement interface was 1.0 and at the stem-cement interface: 0.40. Thirdly, the effects of bone-cement interface rugosity were investigated $\mu_{bc} = 0.4; 0.6; 0.8; 1.0$. For this example, the thickness of the cement mantle was set $4mm$.
2. For *thermal analysis,* we chose the chromium-cobalt implant inserted with cement of uniform thickness. Two interfaces (stem-cement: $\mu_{sc} = 1.0$; cement-bone: μ_{sc} variable) were considered. The distribution of temperature within the cemented bone-implant system was simulated with different cement thickness: $2mm$, $3mm$, $4mm$, and $5mm$. For initial and boundary conditions, the temperature of the implant and the bone is initially set as $37°$. The cement temperature was held uniform and constant at $100°$. The outer surface of the femur was kept at the body temperature $37°$.

Some results

The distribution of temperature and stress were calculated inside bone, cement, and stem, and at the interfaces during implantation. Inside structures, peak of temperature was obviously located in the cement. This peak was minimal ($76°$) when the cement thickness was the lowest $2mm$, and maximal ($90°$) for a cement mantle of $5mm$. At the bone-cement interface, the calculation has shown that temperature increases also with thickness. Stress and temperature were lowest ($50°$) for a cement mantle thickness of $2mm - 3mm$ and maximal ($63°$) for a thickness of $5mm$. Moreover, a greater part of the heat generated by polymerization was absorbed by cement and stem. It is attempting to relate the evolution of bone-cement temperature to the probability of late failure at this interface. It was shown that pressure magnitude varies from $+296MPa$ (tensile) and $-97MPa$ (compression). The existence of tensile regions in the vicinity of the interfaces means the occurrence of residual stress when the cement temperature decreases in the course of time. High residual stress near the interfaces is a probable apparition of microcracks density since the mechanical strengths of cement-bone interface, for instance, are respectively $2.25MPa$ and $1.35MPa$.In addition to temperature field, it was shown that pressure magnitude varies from $+296MPa$ (tensile) and $-97MPa$ (compression). The existence of tensile regions in the vicinity of the interfaces means the occurrence of residual stress when the cement temperature decreases in the course of time. High residual stress near the interfaces is a probable apparition of microcracks density since the mechanical strengths of cement-bone interface, for instance, are respectively $2.25MPa$ and $1.35MPa$.

CONCLUSIONS

Contact problemsare centralto orthopaedic biomechanics because contact is the principalmean for transfering loads from the implant to bone and vice versa. Thermal stresses due to heat generation and/or heat transfer accross the interface may also decrease the reliability of the cemented implant anchorage. The accuracy of calculation at the interfaces is crucial for reliability of numerical methods and results in this domain. This is demanded by the very high precision of the biological environment at the interfaces: sensitivity relative to the range of shear micromotions for fibrous tissue evolution, range of stress stimulus for bone remodelling, range of temperature for bone necrosis. In most cases, the use of augmented Lagrangian methods in contact thermomechanics at the bone-implant interface is justified since mixed penalty-duality methods are known to be more robust than primal penalty methods. Multiplier methods have the great advantages to give exact solutions of the thermocontact models independently on the algorithm parameters.

References

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