EVOLUTIONS OF FRICTION ANISOTROPY AND HETEROGENEITY

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Summary Evolution and reorientation of microstructures take place in sliding surfaces of some polymers and layer-lattice materials. Advanced friction models describe evolutions of friction induced by a curvature of sliding trajectory. The sliding path curvature generates positive and negative additional friction and constraint forces normal to the sliding trajectory.

INTRODUCTION

During friction and sliding, besides surface roughness changes, material sliding surfaces may be subjected to evolution and reorientation of their microstructures. For instance, complex kinematics of sliding initiate microstructural and frictional changes in sliding surfaces of some polymers [1] and in layer-lattice materials as graphite [3] and molybdenum disulphide [2]. In these cases, microstructures of the sliding surfaces reorient themselves from randomly to highly oriented, and from one orientation to other (a self-organisation phenomenon). It may reduce (or increase) friction and wear. There are preferential sliding directions.

Advanced friction models can describe evolutions of frictional anisotropy and heterogeneity induced by the sliding kinematics. Due to this, first-, second- and higher-order descriptions of friction are developed with respect to powers of the sliding path curvature.

MODELS OF ANISOTROPIC AND HETEROGENEOUS FRICTION

Constitutive equations governing the phenomenon of anisotropic and heterogeneous friction are created in the frame of Amontons-Coulomb friction law. By the friction constitutive equation we mean a relation between a friction force vector **t** and a normal pressure N, a sliding velocity unit vector **v** and its derivative d**v**/ds with respect to a one-dimensional parameterization s of the sliding trajectory.

A first-order description

A first-order constitutive equation of the friction force has two independent variables, i.e. the sliding velocity unit vector **v** and its derivative d**v**/ds. It can be defined as a sum of two single-term polynomials as follows

$$\mathbf{t} = -N(\mathbf{C}_1\mathbf{v} + \mathbf{E}_1\mathbf{n}\frac{1}{\rho}), \tag{1}$$

where, according to the Frenet-Serret first formula d**v**/ds = **n**/ρ, **n** is a unit vector normal to the sliding path, ρ is a sliding path radius. Two second-order tensors $\mathbf{C}_1$ and $\mathbf{E}_1$ are equation coefficients (so called parametric tensors). For instance, the tensor $\mathbf{C}_1$ can define friction properties in the sliding surface which form (in geometrical terms): (a) concentric circles, (b) set of radii (i.e. rays emanating from the origin), see [4,5]. The tensor $\mathbf{E}_1$ describes constraints imposed on the motion in the directions tangent and normal to the sliding path. These constraints are functions of the first power of the sliding path curvature. The friction force (1) depends on the sliding direction and on a location of the contact point within the sliding area.

A second-order description

In the second-order formulation, the friction constitutive equation is a polynomial with respect to third-order tensors composed by **v** and **n**/ρ ; they are taken as the independent variables of the friction equation, i.e.

$$\mathbf{t} = -N\{\mathbf{C}_1\mathbf{v} + \mathbf{C}_2(\mathbf{v}^3) + \mathbf{E}_1\mathbf{n}\frac{1}{\rho} + \mathbf{E}_2[(\mathbf{v}^2,\mathbf{n})\frac{1}{\rho} + \ldots + (\mathbf{v},\mathbf{n}^2)\frac{1}{\rho^2} + \ldots + (\mathbf{n}^3)\frac{1}{\rho^3}]\}, \tag{2}$$

with the following notation of the third-order tensors

$$(\mathbf{v}^p,\mathbf{n}^q)\frac{1}{\rho^q} \equiv (\underbrace{\mathbf{v}\otimes\ldots\otimes\mathbf{v}}_{p-copies}\otimes\underbrace{\mathbf{n}\otimes\ldots\otimes\mathbf{n}}_{q-copies})\frac{1}{\rho^q}, \tag{3}$$

where, p = 0,1,2,3 q = 0,1,2,3 and (p + q) = 3. The equation coefficients $\mathbf{C}_2$ and $\mathbf{E}_2$ are tensors of the fourth-order; $\mathbf{C}_2$ defines other types of frictional anisotropy and heterogeneity (e.g. local tetragonal anisotropy), $\mathbf{E}_2$ describes additional friction and motion constraints which depend on first, second and third powers of the sliding path curvature.

Higher-order descriptions

Non-homogeneous friction described with the aid of other field singularities requires friction equations of higher order. Then, the polynomial form of the friction equation contains terms which are contractions of the friction tensors $\mathbf{C}_i, \mathbf{E}_i$, i = 1,2,...,n and the higher-order tensors composed by **v** and **n**/ρ. According to the objectivity axiom only odd order tensors composed by **v** and **n**/ρ may be included in the polynomial, i.e. first, third,...,2n-1. The independent variables are composed in accordance to the rule (3), where p = 0,1,...,2n-1, q = 0,1,...,2n-1 and (p + q) = 3,5,...,2n-1. Tensors $\mathbf{C}_i$ define anisotropy and heterogeneity of friction. Tensors $\mathbf{E}_i$ define effects associated with the sliding motion. In this model, the friction force depends on an arbitrary power of the sliding path curvature.

COMPOSITIONS OF TWO FRICTION ANISOTROPIES AND HETEROGENEITIES

It is assumed that for a given normal pressure N the resultant friction force $\mathbf{t}_{AB}$ at the contact of two surfaces (A and (B) is equal to the product of a "composition coefficient" by the sum of the friction forces obtained for each surface taken separately, i.e.

$$\mathbf{t}_{AB} = \kappa(\overset{(A)}{\mathbf{t}} + \overset{(B)}{\mathbf{t}}), \tag{4}$$

where, κ is the experimental composition coefficient, $\overset{(A)}{\mathbf{t}}$ and $\overset{(B)}{\mathbf{t}}$ are friction forces which correspond to the friction when sliding a third body with isotropic homogeneous properties along the contacting surfaces (A) and (B). The friction forces for surfaces (A) and (B) can be represented by equations (1) and (2). The hypothesis (4) describes changes in friction anisotropy and non-homogeneity when two different sliding surfaces are composed together.

EXAMPLES

The friction constitutive equations proposed in this contribution are completed with illustrative examples. Sliding trajectories of a material point in the surface with complex tribological properties are investigated. Sliding motions are excited by initial velocities. Motion equations are given in the local basis defined by unit vectors tangent and normal to the sliding trajectory {**v**, **n**}. The friction force components in the basis {**v**, **n**} depend not only on the sliding direction but also on a shape of the sliding path. Different types of anisotropic and non-homogeneous friction properties are considered, e.g. the friction properties which form concentric circles in the surface. In this case, there are two types of privileged sliding directions: radial (along radii), concentric circular (along concentric circles). Essential changes of sliding trajectories of the material point are observed for various values of parameters of the friction tensors in the constitutive equations (1) and (2).

CONCLUSIONS

The sliding path curvature generates: (a) additional resistance to sliding (dissipative type forces), (b) constraint forces normal to the sliding path (gyroscopic type forces). It can induce positive and negative additional friction, and it can change essentially the sliding trajectory. Friction cones change cross-sectional shapes in dependence on the sliding trajectory curvature. Rectilinear and curved privileged sliding directions occur in this case.
As in the classical continuum mechanics and thermodynamics, mathematical restrictions on forms and parameters of the friction constitutive equations arise from the objectivity axiom and the Second Law of Thermodynamics (an entropy production inequality), see [4,5].

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