CONTACT OPTIMIZATION PROBLEMS ASSOCIATED WITH THE WEAR PROCESS

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Summary The optimal shape design of contact surfaces has usually been aimed at controlling the contact pressure distribution. However, a much wider class of contact optimization problems can be formulated by maximizing contact force or displacement, torsional moment, or minimizing the rate of dissipation. A special class of optimization problems is considered, namely, the minimization of wear rate depending on both normal pressure and slip velocity. The specific modified Archard wear rule is assumed. The illustrative examples demonstrate the evolution of wear process toward their steady states. It is demonstrated that the wear dissipation power at the contact surface is minimal in the steady state of the wear process and in many cases corresponds to the uniform wear rate. The discretization of the contacting bodies was performed by the displacement based on *p*-version of finite elements assuring fast convergence of the numerical process and accurate specification of geometry for shape optimization, or by half space Boussinesq solution. It is assumed that the displacements and deformations are small, the material of the contacting bodies are elastic.

INTRODUCTION

In optimization problems the contact shape is usually determined in order to minimize the maximal pressure, thus achieving a uniform pressure over some contact portions. The book of Haslinger and Neittaanmaki [1] presents typical formulations of problems of optimal contact shape. An extensive survey of contact pressure optimization problems was presented by Hilding and Klarbring [2]. In Goryacheva's book [3] the contact problems of wearing process are investigated in analytical way. The papers by Páczelt [4] and Páczelt and Szabó [5] provide solutions for 2D and 3D problems in which the contact pressure distribution is partially controlled by minimizing the maximum contact pressure. The present work provides an analysis of evolution of contact surface shape and pressure due to wear process. It is supposed that the friction and wear is isotropic. The idea that wear depends on frictional dissipation was proposed by Mróz and Stupkiewicz [6] for the case of anisotropic friction and wear. It is believed that the optimal shape is achieved in this process when the wear rate becomes uniform within the contact domain. Such uniform wear rate criterion corresponds to the minimum of the dissipation power associated with the wear process. Without the restriction of generality, let us consider the contact problem of two elastic bodies $B_\alpha, (\alpha = 1,2)$ with the usual boundary and loading conditions. The boundary portion $S_c^{(\alpha)}$ will be called the potential zone of contact. In this part of the bodies the shape may be modified. In the normal direction the Signorini contact conditions are valid [7]. The Coulomb dry friction models are investigated. In the analysis of wear problem, usually the elastic portion of relative tangent velocity is much smaller than the rigid body motion induced velocity, thus the effect of elastic component of tangent relative velocity can be neglected in the wear analysis. We shall use this fact in our numerical study of wear process. The temperature effects and heat generated at the frictional interface [7,8] in our investigation is out of consideration. The boundary value problem is solved by the variation principles using modified complementary energy, total potential energy with augmented Lagrangian technique [7]. The contact conditions are checked at the Gauss or Lobatto integration points of the contact elements during the solution process. The oscillations of contact pressure and displacements are minimized by using the node positioning and re-meshing technique described in [9].

Controlling of distribution of contact pressure. In optimization problems [4] we suppose that the bodies are in contact on the whole sub-domain Ω_c of the contact zone $S_c = \Omega$. The contact surface is modified in the way that the following function is held for the contact pressure $p(\mathbf{x}) = c(\mathbf{x})\, p_{\max}$, where the chosen control function must satisfy the condition $0 \le c(\mathbf{x}) \le 1$, and $p_{\max} = \max p(\mathbf{x})$, $\mathbf{x} = [s,t]$, where s and t are surface coordinates in the region Ω. In the sub-domain $\Omega_{nc}\, (\Omega = \Omega_c \bigcup \Omega_{nc})$ the satisfaction of the following inequality is required: $\chi(\mathbf{x}) = c(\mathbf{x})\, p_{\max} - p(\mathbf{x}) \ge 0$ $\mathbf{x} \in \Omega_{nc}$. Usually we use the control function $c(s)$ of class C^1 in the subregion Ω_c.

MINIMIZATION OF WEAR DISSIPATION POWER

Let us consider the wear problem of the punch B_1, translates or rotates with respect to the body B_2 which is constrained and does not undergo rigid body motion. In the case of translatory motion the relative sliding velocity at contact surface S_c v_r is constant, in the case of rotary motion, it is equal $v_r = r\,\omega$, where r denotes the distance of a point on S_c from the axis of rotation. Assume the isotropic wear rule in the form $\dot{w} = \beta\,(\mu\, p_n)^b\, v_r^a = \tilde{\beta}\, p_n^{\,b}\, v_r^a$, where a, b, β are wear parameters, μ, p_n are coefficient friction and contact pressure, $\tilde{\beta} = \beta\,\mu^b$. The shearing stress in the contact surface denoting by τ_n and calculated from the contact pressure by the Coulomb dry friction law $\tau_n = \mu\,p_n$. The wear volume rate is calculated by the next formula $\dot{W} = \int_{S_c} \dot{w}\,dS = \int_{S_c} \tilde{\beta}\;p_n^{\,b}\,v_r^a\,dS$ and the wear dissipation power at the contact surface S_c equals $D_w = \int_{S_c} p_n \dot{w}\,dS = \int_{S_c} \tilde{\beta}\;p_n^{\,b+1}\,v_r^a\,dS$.

P.1 Consider first the stationary motion, $v_r = const$. Assume the contact pressure $p_n(\mathbf{x})$ to satisfy the global equilibrium conditions, for body B_1, we have $\mathbf{f} = -\int_{S_c} \mathbf{n}_c\,p_n\,dS + \mathbf{f}_0 = \mathbf{0}$, $\mathbf{m} = -\int_{S_c} \Delta\mathbf{r}\times\mathbf{n}_c\,p_n\,dS + \mathbf{m}_0 = \mathbf{0}$, where $\mathbf{n}_c$ is the contact normal direction vector, $\mathbf{f}_0$ and $\mathbf{m}_0$ are resultant force and moment of tractions acting on body B_1, $\Delta\mathbf{r}$ is the position vector. Introducing the Lagrange multipliers $\dot{\boldsymbol{\lambda}}_F$ and $\dot{\boldsymbol{\lambda}}_M$, consider the Lagrangian function $L_{D_w} = L_{D_w}\left(p_n, \dot{\boldsymbol{\lambda}}_F, \dot{\boldsymbol{\lambda}}_M\right) = \int_{S_c} \tilde{\beta}\;p_n^{\,b+1}\,v_r^a\,dS + \dot{\boldsymbol{\lambda}}_F\cdot\mathbf{f} + \dot{\boldsymbol{\lambda}}_M\cdot\mathbf{m}$. The stationary condition $\delta L_{D_w} = 0$ now provides the optimal contact pressures distribution and the equilibrium conditions thus $p_n = \left(\dfrac{\dot{\boldsymbol{\lambda}}_F\cdot\mathbf{n}_c + \left(\dot{\boldsymbol{\lambda}}_M\times\Delta\mathbf{r}\right)\cdot\mathbf{n}_c}{(b+1)\,\tilde{\beta}\,v_r^a}\right)^{\frac{1}{b}}$. For calculation of Lagrangian multipliers $\dot{\boldsymbol{\lambda}}_F, \dot{\boldsymbol{\lambda}}_M$ the equilibrium equations must be used. These nonlinear equations may be solved with Newton-Raphson technique.

P.2 Consider now the rotary motion of the punch B_1 with r_i internal and r_e external radius. Now we have $v_r = r\,\omega$, $\dot{w} = \tilde{\beta}\;p_n^{\,b}\,(r\,\omega)^a$. Because the punch has rigid body translation in the direction of rotation axis, we consider only normal force constraint, that is, the Lagrange function takes the form $L_{D_w}^{rot}(p_n, \dot{\lambda}_F) = \int_{S_c} \tilde{\beta}\;p_n^{\,b+1}\,(r\,\omega)^a\,dS - \dot{\lambda}_F\left(\int_{S_c} p_n\,dS - F_0\right)$. Introduce the integral $I_{D_w} = 2\,\pi\int_{r_i}^{r_e} r^{1-\frac{a}{b}}\,dr$, the Lagrangian multiplier equals $\dot{\lambda}_F = \left(\dfrac{F_0}{I_{D_w}}\right)^b (b+1)\,\tilde{\beta}\,\omega^a$ and contact pressure is $p_n = \dfrac{F_0}{I_{D_w}}\,r^{-\frac{a}{b}}$. In this case the wear rate is uniform $\dot{w} = \tilde{\beta}\,\omega^a\left(\dfrac{F_0}{I_{D_w}}\right)^b = const,$ and the wear volume rate can be calculated in a very easy way.

It is very important, that the optimal contact shape is attained when the wear dissipation power is minimal. This shape can be calculated "a priori" using the control technique with the optimal contact pressure which will be applied on the bodies at the surfaces $S_c^{(\alpha)}$ [4].

CONCLUSIONS

It is proved that minimum of the wear dissipation power at the contact surface provides the wear rate characterizing the steady wear process. The minimization of the total dissipation power gives different results dependent on form of rigid body motion and loading condition of contacting bodies. This optimal or steady state shape can be obtained through the optimization procedure based on required pressure distribution or through the modeling of wear process using the assumed wear rule.

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