INFLUENCE OF ACOUSTIC WAVES ON STABILITY OF SLIDING BETWEEN TWO ELASTIC SOLIDS

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Summary Steadily sliding between two contacting elastic half-spaces with Coulomb friction under the action of incident harmonic body waves is studied. Partial stick and separation of the interface is considered. Fourier analysis technique is used to solve the mixed non-linear boundary value problem. Influences of the incident wave on the steady sliding between two solids are discussed in details.

PROBLEM STATEMENT

Consider two contacting elastic half-spaces slide steadily with the speed U_0 under the action of the remote shearing traction $\tau^{\infty} = \pm f_{\mathrm{k}} p^{\infty}$ where f_{k} is the kinetic friction coefficient and p^{∞} is the applied pressure. U_0 is assumed to be far smaller than any wave speed. An incident harmonic plane body wave (P or SV type) with the general form of $\mathbf{u}^{(0)} = C_0 \mathbf{d}^{(0)} \exp[\mathrm{i}k_0(x_1 \sin\theta_0 + x_2 \cos\theta_0 - c_0 t)]$ strikes the interface and is reflected and refracted at the interface. We set a Cartesian coordinate with $x_1 x_2$-plane perpendicular to the front surface of the incident wave as shown in Fig.1. The incident angle is θ_0. The direction of the applied shearing traction τ^{∞} or the sliding speed U_0 makes an angle of φ with respective to x_1-axis, and therefore the components of τ^{∞} and U_0 in x_1- and x_3-direction are $\tau^{\infty}\cos\varphi$, $\tau^{\infty}\sin\varphi$ and $U_0\cos\varphi$, $U_0\sin\varphi$. The indices n=1,2,3,4, which may appear in suffix or affix positions are to distinguish between the reflected and refracted P and SV waves, and n=2,4 to distinguish between the induced SH waves. The purpose of the paper is to examine the stability of the sliding between the solids under the action of the incident waves. To this end, we will seek the steady state solution of the problem. Sub-critical angle incidence is considered.

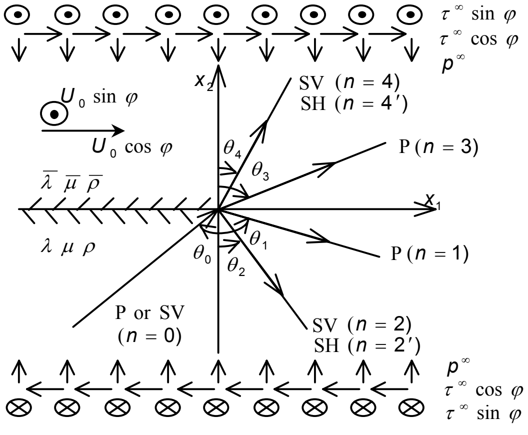

Fig.1 sliding between the solids under the action of the incident

If both applied pressure p^{∞} and sliding speed U_0 is sufficiently small (or equivalently the incident wave is sufficiently strong), partial separation and stick may appear along the interface. Denote the interface traction components by S_j (j= 1,2,3), the relative slip velocity components by V_j (j= 1,3), and the gap by g. Then the boundary conditions in three different regions may be written as follows,

(i) in the separation regions: $S_j = 0, g > 0$;

(ii) in the slip regions: $S_2 < 0, \sqrt{S_1^2 + S_3^2} = f_k |S_2|,\ g = 0, V_1/S_1 = V_3/S_3, \mathrm{sign}(V_j) = \mathrm{sign}(S_j)$;

(iii) in the stick regions: $S_2 < 0, \sqrt{S_1^2 + S_3^2} < f_s |S_2|,\ g = 0, V_1 = V_3 = 0$.

Generally the condition $g = 0$ is replaced by the weaker condition $V_2 = \dot{g} = 0$ with special consideration ensuring that the gap indeed vanishes in the slip and stick regions.

BASIC IDEAS IN SOLUTION OF THE PROBLEM

The difficulties in solving the problem arise from the non-linear boundary conditions that involve inequalities and unknown intervals (i.e. slip, stick and separation regions). Here we state some key ideas:

(*i*) We would look for the steady state solution of the problem. It can be easily proved that Snell's law still holds [1-3]. Therefore we could formulate the problem in a coordinate moving along x_1-axis with the speed of $c_0 / \sin\theta_0$.

(*ii*) We construct the solution by adding the corrective solution $\{\hat{\mathbf{u}}, \hat{\boldsymbol{\sigma}}\}$ to the results for the welded interface case $\{\mathbf{u}, \boldsymbol{\sigma}\}$ (bilateral solution) as in Ref.[1-3]. The interface traction corresponding to the bilateral solution may be expressed as $\mathbf{A}^0 \sin(\eta)$ where $\eta = k_0(x_1 \sin\theta_0 - c_0 t)$.

(*iii*) Due to the nonlinear nature of the problem, the corrective solution may be expressed as the Fourier series containing all higher frequencies [1-3]. The by considering the continuity of the interface traction, the following important local relation can be obtained,

$$\mathbf{S}(\eta) = \boldsymbol{\tau}^{\infty} + \mathbf{A}^0 \sin\eta - \boldsymbol{\lambda}[\mathbf{V}(\eta) - \mathbf{U}],$$

where $\boldsymbol{\tau}^{\infty} = \{\tau^{\infty}\cos\varphi, -p^{\infty}, \tau^{\infty}\sin\varphi\}$, $\mathbf{U} = \{U_0\cos\varphi, 0, U_0\sin\varphi\}$ and $\boldsymbol{\lambda}$ is a matrix relevant to the material properties and the incident wave.

(*iv*) Based on the above relation we look for the solutions with generalized forms in three different regions — slip, stick and separation. The locations and boundaries of the three regions cannot be determined up to now. They must be determined by using an iterative method for particular examples. However, we could anticipate the possible arrangement of the three regions and discuss the conditions for appearing of partial stick and separation. For details, we refer to Refs.[1,2].

(*v*) With the generalized solutions for three different regions in hand, we discuss the stability of the sliding for two situations: coupling ($\varphi \neq 0$) and uncoupling ($\varphi = 0$) of in-plane and anti-plane wave motions, with or without partial stick/separation. The conditions for the appearance of partial stick or separation are presented.

(*vi*) To determine whether the steady state solution exists, the following equations should be checked,

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} V_1(\xi)d\xi = U_0\cos\varphi; \quad \frac{1}{2\pi}\int_{-\pi}^{\pi} V_3(\xi)d\xi = U_0\sin\varphi.$$

CONCLUSIONS

(*i*) For the general case with coupling between in-plane and anti-plane wave motions, no finite steady state solution exists when the loads and sliding speed are fixed. The two solids cannot slide with the original speed unless we adjust the applied shearing tractions in both directions to reduce the resultant value to a lower level than the friction (then we recover the case studied in Refs.[1,2]).

(*ii*) For the special case without coupling between in-plane and anti-plane wave motions, the two solids can slide with the original speed when no partial stick appears at the interface, but cannot when partial stick takes place. To keep the solids sliding with the original speed, one should reduce the applied shearing traction to a lower level. This is the case studied in Ref.[3].

(*iii*) It seems that the incident waves will reduce the apparent friction of the interface especially when partial stick appears.

(*iv*) Partial separation at the interface does not affect the global sliding speed.

References

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