CONTACT FORCE DISTRIBUTION AMONG PINS OF TROCHOID TRANSMISSIONS

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Summary: the paper is concerned with research of trochoid transmissions including cycloid speed reducer, trochoid pump and linear motion trochoid drive. Cycloid speed reducer is a statically indeterminable system. Therefore it is required using a deformation equation while analyzing force distribution. Besides significant influence on contact force is caused by machining tolerances. In the paper the clear mathematical models of cycloidal speed reducer with account machining tolerances, methods for gap and contact force distribution determination and computer-aided modeling results on these approach basis are presented. The simple practical design equation for backlash calculation, expressing relationship between drive parameters (eccentricity, gear ratio, transmission angle), machining tolerance and backlash has been developed. The three-dimensional model of the new linear motion trochoid drive has been presented.

The cycloid drive has high efficiency and small size under wide range of gear ratio (11 to 111) in a single stage. These advantages allow to apply it widely in industry, especially in machine tools and robotics. The crucially important problem for a given application is backlash determination***, the angle trough which the output shaft can rotate when the input shaft is held fixed***, for precision positioning predictions of a working body and load capacity of the speed reducer components. Backlash is caused by existence of machining tolerances, the later being inevitable due to precision of machine, prevention of jamming condition and application of lubricants. Machining tolerances also influence on the contact force distribution among pins. In a few references the problem has been considered. Yang and Blanch developed an analytical model of the cycloid drive with machining tolerances using complex vectors and homogeneous transformations [1]. However one is difficult while its computer realization.

As far as the cycloid speed reducer is produced with a high class of accuracy and the selective assembly is provided, it is possible to consider clearances arising from manufacturing error for all pins as identical.

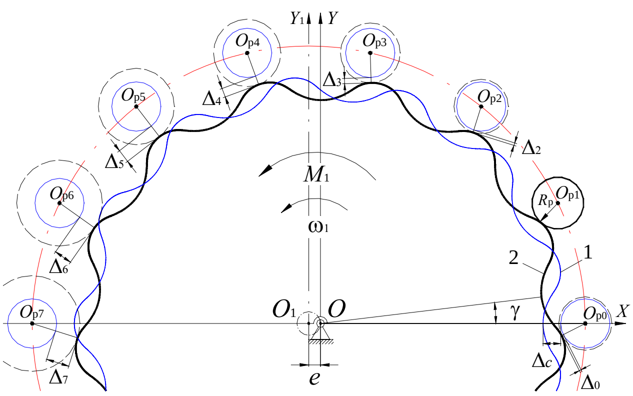

Figure 1 Positions of the cycloid wheel

O_1 – the center of the coordinate system attached to the pin gear; O – the center of the coordinate system attached to the planet wheel; O_{pi} – a circular center of the pin i; Δ_i – the original normal gap between the pin i and the appropriate lobe after eliminating backlash; e – eccentricity.

Let the planet wheel come after assembly into position in relation to the pins shown in Fig.1 by a thin line 1, as if its profile were ideal, with angle α of the eccentric rotation equal to 0. Under the load, applied to output shaft, cycloid gear is turned around own axis O with the angle γ (this position is shown in Fig.1 by a bold line 2) and eliminates the gap between lobe and the first contact pin surfaces. The profile of the planet wheel with account machining tolerance Δ_c can be modelled as equidistant to the ideal epitrochoid profile with a constant spacing equal to Δ_c:

$$\left.\begin{aligned}
X(\varphi) &= R_{pg}\cdot\cos\left(\varphi-\alpha/Z_1\right)-e\cdot\cos\left(Z_2\varphi-\alpha/Z_1\right)-\\
&-(R_p+\Delta_c)\cdot\cos\left(\varphi+\beta-\alpha/Z_1\right)+e\cdot\cos(\alpha)\\
Y(\varphi) &= R_{pg}\cdot\sin\left(\varphi-\alpha/Z_1\right)-e\cdot\sin\left(Z_2\varphi-\alpha/Z_1\right)-\\
&-(R_p+\Delta_c)\cdot\sin\left(\varphi+\beta-\alpha/Z_1\right)+e\cdot\sin(\alpha)
\end{aligned}\right\}$$

Here R_{pg} – pitch radius of the pin gear; R_p – pin radius; Z_1 – the number of lobes of the cycloid disc; Z_2 – the number of pins ($Z_2=Z_1+1$); φ – angle of the epitrochoid generation, varies from 0 to 2π; X, Y – Cartesian coordinates of the planet wheel profile in term of parameter φ relative to the fixed point O_1 under the eccentric rotation angle equal to α.

Cycloid planetary drive is a statically indeterminable system. Therefore it is required using a deformation equation while analyzing force distribution [2]. Under driving force F_e acting, applied to planet wheel from eccentric side, other pins come into contact with lobes caused by their elastic deformation. Owing to this fact the cycloid disc is rotated a little angle $\Delta\varphi$. The scheme for elastic deformation definition is presented in Figure 2.

The distance which theoretical contact point C_i on the tooth profile moves perpendicularly to the radius r_i is equal to $r_i\Delta\varphi$ from $\Delta OC_iC'_i$ (since the angle $\Delta\varphi$ is small). Then the elastic deformation δ_i value is expressed from $\Delta C_iC'_iA$ as:

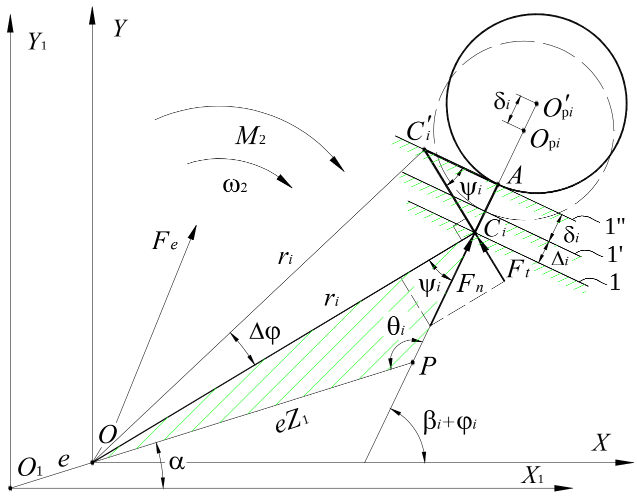

Figure 2 Elastic deformation determination diagram

$$\delta_i = \Delta\varphi \cdot r_i \cdot \sin\psi_i - \Delta_i \tag{1}$$

Here i – pin number. The equation (1) can be simplified while considering ΔOC_iP:

$$r_i \cdot \sin\psi_i = eZ_1 \cdot \sin\theta_i \tag{2}$$

$$\theta_i = \pi - (\beta_i + \varphi_i - \alpha) \tag{3}$$

Here r_i – distance between the planet wheel rotation center O and contact point C_i.

In the limits of a linear elastic deformation, the normal force F_{ni} is proportional with the elastic deformation:

$$F_{ni} = K \cdot \delta_i \tag{4}$$

During the engagement process, driving force F_e is transmitted to the pins at the contact points. Both lobes and pins are deformed under the load. As far as lobes of the cycloid gear are in compression they are not subjected to share force. The pin is considered as a supported beam with round section under uniformly distributed loading. The value of K can be obtained from equation:

$$F_n = \left(\frac{b \cdot EJ}{A_1 x^3 - A_2 (x-a)^4 + A_3 (x-a-b)^4 + A_4 x - A_5} \right) \cdot \delta_b \tag{5}$$

Here $A_{1...5}$ – constant. Once the gap distribution Δ_i and rotational elastic deformation $\Delta\varphi$ have been calculated, the force distribution among pins can be obtained. The sign of Eq.(1) indicates whether a pin is in a loaded state *(+)* or unloaded state *(-)*. The comparison diagrams of the force distribution among pins with account tolerance and without are presented in Figure 3. Graphical presentation of the contact force distribution is presented in Fig.4.

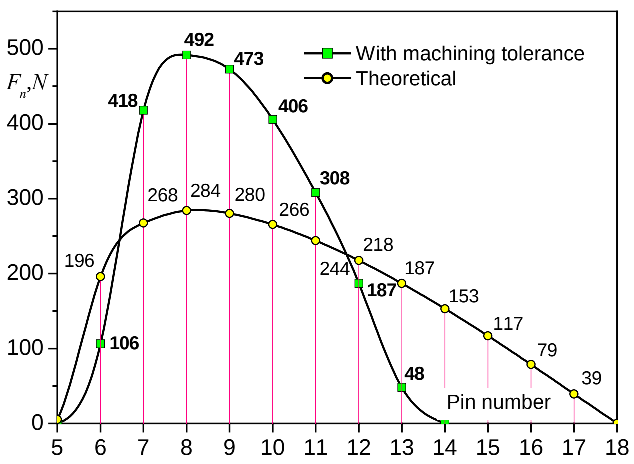

Figure 3 Diagrams of the force distribution among

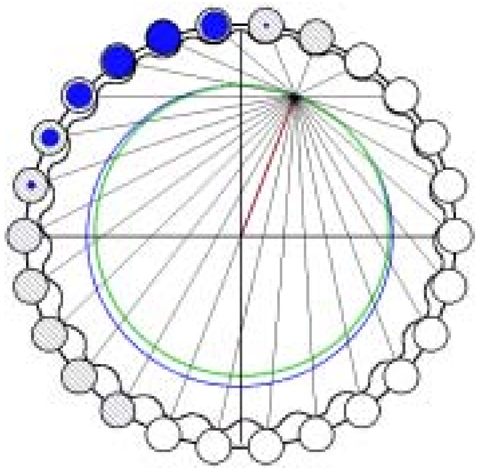

Figure 4

CONCLUSIONS

Obtained results show the necessity for the account machining tolerances under cycloid drive design. The approach for contact force calculation has been presented and approved by comparison of experimental and theoretical results.

References

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