ELECTRIC-MECHANICAL BEAM-TO-BEAM CONTACT

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Summary. In this paper the formulation of an electric-mechanical beam-to-beam contact element is presented. Beams with circular cross-sections are assumed to get in contact in a point-wise manner and with clean metallic surfaces. The voltage distribution is influenced by the contact mechanics, since the current flow is constricted through small contacting spots. Therefore the solution is governed by the contacting areas and hence by the contact forces. As a consequence the problem is semi-coupled with the mechanical field influencing the electric one. The electric-mechanical contact constraints are enforced with the penalty method within the Finite Element technique. The virtual work equations for the mechanical and electric fields are written and consistently linearized to achieve a good level of computational efficiency with the FE method. The equations set is solved with a monolithic approach.

INTRODUCTION

The pure mechanical beam-to-beam contact has been studied recently in [1] and [2]. In some advanced technology problems, such as nuclear and astronautics technology, electronic component mechanics, superconducting cables and coils, it is necessary to combine mechanical contact with other fields, such as the electric or thermal one. This paper is the first attempt to formulate a beam-to-beam contact element for the coupled electric-mechanical field. The electric part of the contact potential is taken in the same way as in [3], where it is described for the electric-mechanical contact between 2D continuum bodies using the node-to-segment approach. Concerning the contact area, taking into account the observations from [4] a long constriction is assumed, when considering a circular contact spot with a radius very small with respect to the beam radius. The contact area is calculated with Hertz contact theory for elastic bodies. In this way the problem is semi-coupled, the voltage distribution being affected by the mechanical contact areas. The virtual work equations for the mechanical and electric fields are written and consistently linearized to achieve a good level of computational efficiency with the FE method. The equations set is solved with a monolithic approach.

MECHANICAL CONTACT

For the clarity just two beams with circular cross-sections with radii r and $\bar{r}$ are considered. The extension to the case with more beams is straightforward. Beams are assumed to get in contact with clean metallic surfaces in a point-wise way. The key issue is to locate the closest points on the beams which are the potential candidates for the contact. To this end the orthogonality conditions are used involving the current position vectors of axis points for both beams: $\mathbf{x}$ and $\bar{\mathbf{x}}$ being the functions of respective local co-ordinates ξ and $\bar{\xi}$

$$\begin{cases} (\bar{\mathbf{x}} - \mathbf{x}) \cdot \mathbf{x}_{,\xi} = 0 \\ (\bar{\mathbf{x}} - \mathbf{x}) \cdot \bar{\mathbf{x}}_{,\bar{\xi}} = 0 \end{cases}$$

With the potential contact points known the normal gap is defined as

$$g_N = \|\bar{\mathbf{x}} - \mathbf{x}\|$$

and depending on its value the possible contact is determined with the geometrical check [1], [2].
In the frame of the penalty method the contact contribution to the virtual work for one active contact pair takes the form

$$\delta\Pi_m = \varepsilon_N \cdot g_N \cdot \delta_u g_N$$

where ε_N is the penalty parameter. Newton-Raphson method requires its linearization with respect to mechanical and electric unknowns

$$\Delta_u \delta_u \Pi_m = \varepsilon_N \cdot \delta_u g_N \cdot \Delta_u g_N + \varepsilon_N \cdot g_N \cdot \Delta_u \delta_u g_N \qquad \Delta_V \delta_u \Pi_m = 0$$

The first linearization represents the standard mechanical contact which will not be discussed here, see [1] or [2] for the details. The second one is zero since no dependence of the mechanical terms on the electric variables is considered.

ELECTRIC CONTACT

Having determined the location of the closest points their respective voltages V and $\bar{V}$ are computed linearly from the beam nodes voltages and are used to define the voltage gap

$$g_V = \bar{V} - V$$

With this in hand the electric contact contribution for one active pair takes the form

$$\delta\Pi_e = I \cdot \delta_V g_V$$

where I denotes the electric current intensity flowing through the contact area.

In our case of beam-to-beam contact it is assumed as a first approximation that the contacting area is circular with radius a much smaller than the radii r and $\bar{r}$ of the beams. In this way the conducting area is very small and with clean metallic surfaces the contact resistance is only the constriction resistance [4]

$$R = \frac{r}{2a}$$

where r denotes the material resistivity. Assuming an elastic behaviour of the contact area Hertz theory can be considered, and the contact radius is

$$a = \left(\frac{3\varepsilon_N g_N r^*}{4E^*} \right)^{\frac{1}{3}}$$

where the mean radius $r^* = \frac{r\bar{r}}{r+\bar{r}}$ and the mean Young's modulus $E^* = \frac{E\bar{E}}{\bar{E}\left(1-\nu^2\right)+E\left(1-\bar{\nu}^2\right)}$ have been introduced.

The linearization of the electric contact contribution with respect to all variables has the following form

$$\Delta_u \delta_V \Pi_e = \frac{\partial I}{\partial g_V} \cdot \Delta_u g_V \cdot \delta_V g_V + \frac{\partial I}{\partial g_N} \cdot \Delta_u g_N \cdot \delta_V g_V + I \cdot \Delta_u \delta_V g_V$$

$$\Delta_V \delta_V \Pi_e = \frac{\partial I}{\partial g_V} \cdot \Delta_V g_V \cdot \delta_V g_V$$

The first variation and the linearization of the kinematical and electric variables can be expressed as

$$\delta_u g_N = \mathbf{n}^T \left(\delta\bar{\mathbf{u}} - \delta\mathbf{u} \right)$$

$$\Delta_u g_V = \bar{V}_{,\bar{\xi}} \Delta\bar{\xi} - V_{,\xi} \Delta\xi \qquad \delta_V g_V = \delta\bar{V} - \delta V \qquad \Delta_u \delta_V g_V = \delta\bar{V}_{,\bar{\xi}} \Delta\bar{\xi} - \delta V_{,\xi} \Delta\xi$$

where the unit normal vector is defined as $\mathbf{n} = \frac{\bar{\mathbf{x}} - \mathbf{x}}{\left\| \bar{\mathbf{x}} - \mathbf{x} \right\|}$.

FE DISCRETISATION

The developed beam-to-beam electric-mechanical contact finite element corresponds to the following vector of nodal unknowns

$$\mathbf{q}_{(28\times1)} = \left\{ \bar{\mathbf{u}}_M{}^T, \mathbf{u}_M{}^T, \bar{\mathbf{V}}_M{}^T, \mathbf{V}_M{}^T \right\}$$

where vectors $\mathbf{u}_M$ and $\mathbf{V}_M$ represent the nodal displacements and nodal voltages, respectively of both beam elements within which the contact points lie

$$\mathbf{u}_M = \left\{ \mathbf{u}_1{}^T, \mathbf{u}_2{}^T \right\} = \left\{ u_{x1}, u_{y1}, u_{z1}, \varphi_{x1}, \varphi_{y1}, \varphi_{z1}, u_{x2}, u_{y2}, u_{z2}, \varphi_{x2}, \varphi_{y2}, \varphi_{z2} \right\}^T \qquad \mathbf{V}_M = \left\{ V_1, V_2 \right\}^T$$

The corresponding tangent stiffness matrix and residual vector have the following general form

$$\mathbf{K} = \begin{bmatrix} \mathbf{K}_{mm} & \mathbf{0} \\ \left(\mathbf{K}_{me1} + \mathbf{K}_{me2} + \mathbf{K}_{me3} \right) & \mathbf{K}_{ee} \end{bmatrix}_{(28\times28)} \qquad \mathbf{R} = \begin{bmatrix} \mathbf{R}_m \\ \mathbf{R}_e \end{bmatrix}_{(28\times1)}$$

where indices m and e represent the mechanical and electric terms, respectively.

CONCLUSIONS

The tangent stiffness matrix and residual vector for the electric-mechanical beam-to-beam contact finite element are presented. They are expressed in terms of the corresponding vector of nodal displacements and voltages. In this sense they are easy to be added to any FE package. The performance of the formulation will be confirmed by the numerical examples.

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