AN NATURAL HYBRID-MIXED MODEL WITH SHEAR PROJECTION FOR CURVED SHELLS

Rezak Ayad

Université de Reims Champagne-Ardenne, Groupe Mécanique Matériaux et Structures (GMMS, EA 2617)
ESIEC – Esplanade Roland Garros, BP1029, 51686 Reims

Summary A new natural hybrid-mixed variational approach, applied to a degenerated 4-node shell finite element, is presented. The membrane part is natural hybrid with an improved stress field. The bending part is mixed based on the classical Hellinger-Reissner principle. The transverse shear part is partially hybrid with the satisfaction of two 3D-equilibrium equations. To eliminate the shear locking, the Assumed Natural Strain method is used for approximating shear strains. The results of a pinched cylinder are presented.

INTRODUCTION

The NHMiSP model (Natural Hybrid-Mixed with Shear Projection) [1] is a natural mixed-hybrid approach. It's based the Hellinger-Reissner variational principle, with a particular representation of membrane, bending and transverse shear parts. The membrane part is based on a natural hybrid formulation based on the Pian's approach [2]: the contravariant stresses $\sigma^{\xi\xi},\sigma^{\eta\eta},\sigma^{\xi\eta}$ are approximated in the covariant co-ordinate system ξ,η. Good accuracy on displacements and stresses are obtained indeed. The bending part is based on the classical Hellinger-Reissner principle, in which the bending stresses are approximated using a bilinear lagrange interpolation: twelve bending parameters are then locally eliminated by a static condensation. A mixed-hybrid form is partially used for the shear virtual work ie the two bending equilibrium equations in terms of stresses are satisfied, introducing a new relation between the bending and the shear parameters. The static condensation will be used only for the bending parameters and the computational cost is then reduced. In order to eliminate the shear locking problem which is observed in the standard hybrid-mixed shell models, the Assumed Natural Strain method (ANS)[3] is adopted for defining the transverse shear strains. The NHMiSP model has also the advantage to not consider bubble functions for its stability, making it less expensive than the classical hybrid-mixed models.

THEORETICAL ASPECTS OF THE NHMiSP4 MODEL

The three variational forms for membrane, bending and shear parts are given as follows [1]:

$$W_m^e = h\int_{A^e}\left(<\varepsilon_0^*>\{\sigma_0\}+<\sigma_0^*>\{\varepsilon_0\}-<\sigma_0^*>[H]^{-1}\{\sigma_0\}\right)dA \quad : \text{ membrane} \tag{1}$$

$$W_f^e = \frac{h}{3}\int_{A^e}\left(<\varepsilon_1^*>\{\sigma_1\}+<\sigma_1^*>\{\varepsilon_1\}-<\sigma_1^*>[H]^{-1}\{\sigma_1\}\right)dA \quad : \text{ bending} \tag{2}$$

$$W_c^e = \frac{h^2}{6}\int_{A^e}\left(<\gamma_0^*>\{div[\sigma_1]\}+<div[\sigma_1^*]>\{\gamma_0\}-\frac{h}{5}<div[\sigma_1^*]>[G]^{-1}\{div[\sigma_1^*]\}\right)dA \quad : \text{ shear} \tag{3}$$

$\{\varepsilon_0\}$, $\{\varepsilon_1\}$ et $\{\gamma_0\}$ are respectively the vectors of membrane, bending and shear strains, $\{\sigma_0\}=\{\sigma_{x0},\sigma_{y0},\sigma_{xy0}\}$ the membrane stress vector and $\{\sigma_1\}=\{\sigma_{x1},\sigma_{y1},\sigma_{xy1}\}$ the bending stress vector. The transverse shear stress vector is defined in terms of the vector $\{div[\sigma_1]\}$ which appears in the shear virtual work W_c^e (3). $\{div[\sigma_1]\}$ derives from the use of the two first equilibrium equations in the local cartesian system $[t_1,t_2,n]$ (figure 1). The membrane virtual work W_m^e (1) uses a hybrid form with a complete approximation field for the membrane stresses $\{\sigma_0\}$. This approximation allows to get an accurate element from regular to distored meshes. Adapted for a 4-node curved or degenerated shell element with 6 dofs par node (figure 1), the Natural Hybrid Mixed with Shear Projection (NHMiSP) model has considered 5 membrane parameters and 12 bending parameters which are eliminated by static condensation, a bilinear interpolation of the kinematic variables w,βx,βy and 2*2 gauss points for the exact integration of all matrices.

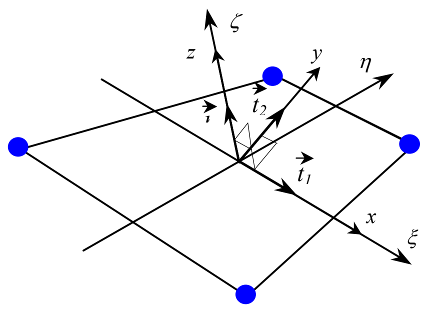

Figure 1. The degenerated natural hybrid-mixed shell element NHMiSP4

NUMERICAL EXAMPLE

Pinched cylinder with diaphragms

Considered as a hard test, a short thin cylinder (L/R=2, R/h=100, L/h=200) with two rigid diaphragms is submited to two unit opposite loads P (figure 2). Results of deflection W_C under load P are presented on figure 3 in terms of finite element meshing N (N=2 to 15 on edges AB and AD). They are given for three known elements (the present model NHMiSP4, MiSP4-Q4 proposed in our previous work [4] and the Bathe &al element MITC4[3]) and compared with a reference solution given by Lindberg &al (1969) [5] : W_{ref} =-$W_C Eh/P$= 164.24. A quite rapid convergence to W_{ref} is given by NHMiSP4 compared with that of MiSP4-Q4 and MITC4 elements, because both membrane and bending behaviours are well represented in our model NHMiSP4, respectively by the natural hybrid and the hybrid-mixed formulations.

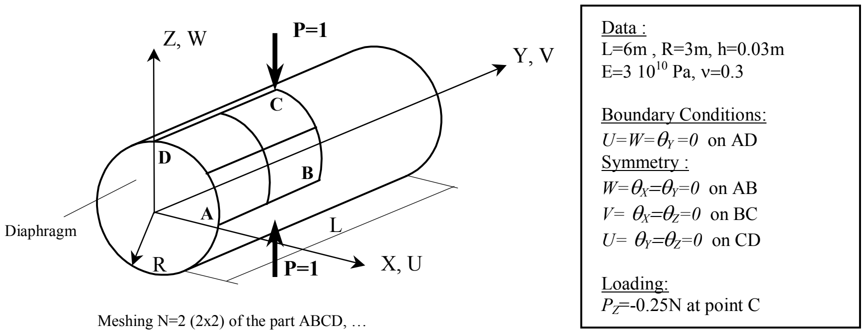

Figure 2. Pinched cylinder with diaphragms

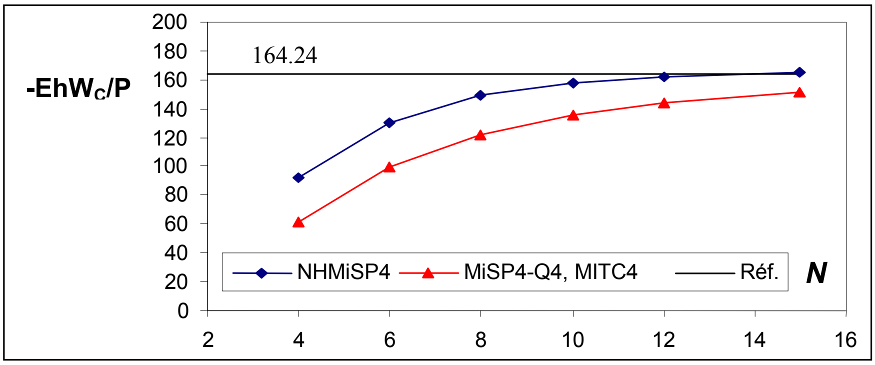

Figure 3. Pinched cylinder with diaphragms. Convergence of deflection W_C

CONCLUDING REMARKS

A new hybrid-mixed degenerated shell finite element with shear effects is presented : the NHMiSP4 model. It naturally takes warpage of shells into account Its natural hybrid representation of the membrane part and its mixed-hybrid representation of the bending/shear part with assumed natural shear strains make it simple and very accurate. NHMiSP4 performs well displacements and stresses of many known hard tests as the twisted 3D beam, pinched cylinder and hemispherical shell etc. An extended version of the present model for multilayered composite shells is actually developed within the context of a Phd thesis at the university of Reims [6].

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