A NEW TRIANGULAR ELEMENT FOR THE ANALYSIS OF COMPOSITE PLATES

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Summary A triangular element based on Mindlin's plate theory is developed for the analysis of composite plates. It has six nodes, each containing five degrees of freedom: in-plane displacements u, v, a transverse displacement w, and rotations of the normal θ_x and θ_y. The key concept lies in assuming the shear rotations γ_x and γ_y as independent field variables instead of θ_x, θ_y. The element passes the patch test and is free from spurious modes or locking. Examples on composite plates are solved which prove the efficacy of the element.

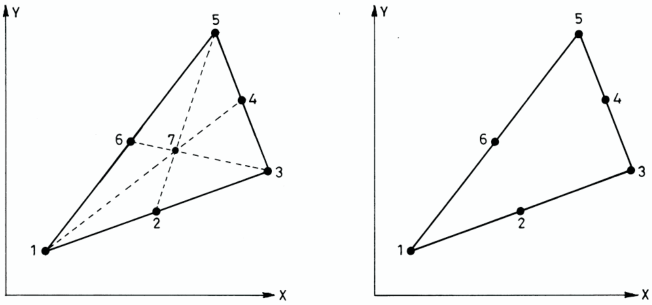
INTRODUCTION

In the last forty years a number of elements have been developed for the analysis of plate bending problems. None so far has been accepted as the single best and suitable for the entire range of problems. Research continues and various manifolds are presently being reconnoitred to develop a general purpose, efficient element.

The earlier attempts with Kirchoff's hypothesis gave incompatible elements, as the satisfaction of C_1 continuity across elements was considerably difficult. Development of effective triangular elements was much sought after, as they has the advantage of representing arbitrary plate shapes in a suitable mesh. But very little light of success was met with. A shift of attention among researchers to Mindlin plate elements brought down the compatibility criteria to the C_0 level. A host of conforming elements were developed in this regime which modelled the rotations of the normal θ_x and θ_y as independent field variables. These elements brought forth a new problem of shear locking. Techniques like reduced integration were proposed to get rid of it, but as a consequence hourglass modes appeared. They could subsequently be stabilised, but the search for alternatives has not yet.

A plate bending element based on Reissner-Mindlin's theory is developed for the analysis of laminated composite plates. The element is triangular with six nodes in the final stage— three at the vertices and three at the mid-sides of the triangle. Each node possesses the five usual degrees of freedom *viz.* two in-plane displacements u and v, a transverse displacement w, and two rotations of the normal θ_x $(= \frac{\partial w}{\partial x} + \gamma_x)$ and θ_y $(= \frac{\partial w}{\partial y} + \gamma_y)$. A seventh node at the centroid having the degrees of freedom w, θ_x and θ_y is primarily assumed but later condensed out to give the element a simple, elegant form.

The cardinal concept lies in assuming the *shear rotations* γ_x and γ_y as *independent* field variables represented by complete linear polynomials. The rotations due to bending $\frac{\partial w}{\partial x}$ and $\frac{\partial w}{\partial y}$ are obtained directly from the derivatives of w (which is approximated by a complete quartic polynomial). This unique feature of the element, on one hand, imposes the requirement of C_1 compatibility (as second order derivatives of w appear in the variational equation), which is kept unfulfilled here. But the advantages are probably surpassing. Firstly it enables a quartic displacement (and cubic rotations) to be modelled with relatively fewer degrees of freedom, thus achieving a high order of accuracy with greatly reduced computation. Moreover, the correct hierarchy of deflection, rotation, moment and shear thus achieved helps to prevent shear locking. As a backup, the element passes the patch test for constant moment, shear, and twist, and is free from spurious modes. And finally, it gives excellent results in case of examples on composite plates.



(a) (b)

Figure 1. Nodes of the proposed element (a) *before* and (b) *after* condensation

NUMERICAL EXAMPLES

A couple of examples have been solved to show the performance of the element in case of (i) *sinusidal* and (ii) *uniform* loading distributions. Two different ply-orientations have been used in the two examples. Each example has been worked out for a series of thickness ratios and mesh divisions, and comparison has been made with available results in standard literature. The material properties used in the examples are :

$$E_1 = 25\,E_2, \quad G_{12} = G_{13} = 0.5\,E_2, \quad G_{23} = 0.2\,E_2, \quad \nu_{12} = 0.25.$$

Simply supported square symmetrically laminated plate subjected to sinusidally distributed load

The problem of a square cross-ply $(0/90/0)$ laminated plate is studied for three different values of span to thickness ratios (a/h). The plate is simply supported on all four sides and subjected to a sinusidally distributed load of amplitude p_0. The intensity of load at any point is given by $p = p_0 \sin(\pi x/a) \sin(\pi y/a),\ x, y \in [-\frac{a}{2}, \frac{a}{2}]$. Utilising structural symmetry a quarter of the plate is modelled. The analysis is done with six different mesh sizes. The deflection obtained at the plate centre is presented in Table 1 with the analytical solution of Reddy [1]. A high rate of convergence of the present results is observed and excellent agreement with the results of Reddy [1] is revealed, even for a very thick plate with $a/h = 4$.

Table 1. Deflection $1000wE_2h^3/(p_0a^4)$ at the centre of a simply supported cross-ply $(0/90/0)$ square laminated plate subjected to sinusoidally distributed load of amplitude p_0

a/h	Present Element						Reddy [1]
	2×2	3×3	4×4	6×6	8×8	10×10	
4	17.861	17.794	17.777	17.766	17.762	17.760	17.76
10	6.8021	6.7393	6.7188	6.7044	6.6994	6.6971	6.69
100	4.4427	4.3831	4.3630	4.3487	4.3436	4.3413	4.34

Simply supported square anti-symmetric laminated plate subjected to uniformly distributed load

A square cross-ply $(0/90)$ laminated plate, simply supported at its four sides and subjected to uniformly distributed load of intensity p is analysed by the proposed element. A quarter of the plate is modelled for four mesh sizes and four different values of span to thickness ratio (a/h). The element is found to be free from shear locking in the case of a very thin plate with $a/h = 10000$. The deflection obtained at the plate centre is presented with the results of Reddy [1] and Kabir [2] in Table 2. The results obtained by Reddy [1] are based on an analytical solution while Kabir [2] has analysed the plate using a three-node triangular element developed by him, and an analysis package *(NISA)* implementing a four-node quadrilateral element. Results obtained by the proposed element show good convergence to the other results.

Table 2. Deflection $1000wE_2h^3/(pa^4)$ at the centre of a simply supported cross-ply $(0/90)$ square laminated plate subjected to uniformly distributed load of intensity p

a/h	Present Element				Reddy [1]	Kabir [2]	*NISA* [2]
	4×4	6×6	8×8	10×10			
10	19.489	19.478	19.474	19.472	19.04	18.95	19.46
48	17.083	17.073	17.069	17.067	17.05	17.00	16.96
1000	16.973	16.963	16.960	16.958	16.95	16.85	16.94
10000	16.972	16.963	16.959	16.958	16.95	16.85	16.94

CONCLUSION

A triangular element based on Mindlin's theory is developed and applied to the analysis of laminated composite plates. The element has six nodes, each containing the five usual degrees of freedom. Performance of the element is tested with two simple problems involving different loading conditions and ply orientation. Excellent results are obtained for thick plates, as well as very thin plates, the latter showing no signs of shear locking. The element also does not have any spurious mode and the patch test is passed successfully. Based on these observations, the element appears highly prospective for the analysis of composite plates.

ACKNOWLEDGMENT

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