A DOUBLY CURVED ELEMENT FOR LAMINATED COMPOSITE SHELLS UNDERGOING FINITE ROTATION

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Summary A finite element formulation via tensor notation for geometrically exact nonlinear laminated composite shells is presented. The present 5/6-DOF shell theory is based on total Lagrangian concept. The two parametric incremental material rotation vector is considered orthogonal to the shell director. To improve the performance of the element EAS combined with ANS scheme is employed.

INTRODUCTION

In present days the curved structural elements made of composite materials have become the optimum structural elements having various attractive features. In this paper an effort has been made to develop a consistent shell element based on a geometrically exact stress resultant based shell theory where the shell director may undergo finite rotation. The present finite element model is based on curvilinear co-ordinates and is quite different from the usual isoparametric concept, in which the shell equations are formulated in global Cartesian co-ordinates with the same interpolation functions for both geometry and displacement fields. The surface intrinsic parameters (e.g., first/second fundamental forms, Christoffel's symbols) are evaluated exactly because the approximate description of the shell geometry causes effects similar to initial imperfections and therefore may falsify the numerical calculation of the sensitive problems. In contrast to the view [1], the present investigation shows that the total Lagrangian formulation can handle arbitrarily large rotations, since the discrepancy emerges when priori kinematic approximations are made to simplify element derivations. To describe the director deformation a constrained two parametric rotation vector along with a stretch parameter has been taken. This type of parametrization enjoys the additive update procedure within any load increment iteration.

Kinematic assumptions

For the definition of two-dimensional displacement variables it is assumed that the points lying in the direction of the undeformed shell mid-surface normal vector $\overset{o}{\mathbf{a}}_3$ remain straight but suffer a constant stretch. Thus the displacement vector

$$\mathbf{U} = \mathbf{R} - \overset{o}{\mathbf{R}} = \mathbf{r} - \overset{o}{\mathbf{r}} + X^3 \left(\mathbf{a}_3 - \overset{o}{\mathbf{a}}_3\right) = \mathbf{u} + X^3 \widetilde{\mathbf{w}} \tag{1}$$

where $\overset{o}{\mathbf{a}}_3$ is a unit vector which shows the X^3-direction of the deformed shell continuum. Now one may think of another independent rotation vector $\boldsymbol{\theta} \in \mathbb{R}^3$(pseudo vector) which rotates the unit vector $\overset{o}{\mathbf{a}}_3$ to $\mathbf{R}(\boldsymbol{\theta})\ \overset{o}{\mathbf{a}}_3$ and one decomposed director displacement vector $\lambda\ \overset{o}{\mathbf{a}}_3$due to the stretch parameter λ, such that the total director displacement vector becomes

$$\widetilde{\mathbf{w}} = \mathbf{a}_3 - \overset{o}{\mathbf{a}}_3 = \mathbf{R}(\boldsymbol{\theta})\ \overset{o}{\mathbf{a}}_3 + \lambda\ \overset{o}{\mathbf{a}}_3 - \overset{o}{\mathbf{a}}_3 \tag{2}$$

The finite element analysis procedure is accordingly developed.

ILLUSTRATIVE EXAMPLES

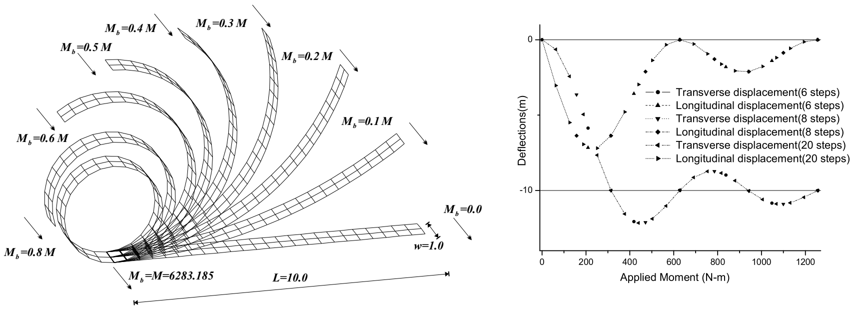
The performance of the present element has been verified with three benchmark problems, as presented below. However, results are also generated for different composite shells, which have not been reported here

Roll-up of a clamped cantilever

An initially flat shell clamped on one end is subjected to a concentrated moment on the other end. This classical problem is used as a benchmark problem for large deformation analysis [2]. According to the Euler beam theory, the analytical solution for a beam rolled up into a circular arc of radius r is given by the formula $1/r = M/EI$, where $M$is the applied end moment. For $M_0 = 2\pi EI/L$ (where L is the length of the beam), the beam rolled up into a circle.The geometric and material properties used for this present problem are $E = 12 \times 10^6 N/m^2, \nu = 0.0, L = 10.0m$, width= $1.0m$ and thickness= $0.1m$. Figure 1(a) depicts several deformed configuration with 9-noded(1×12) elements during loading. Figure 1(b) shows the excellent convergence behavior of the present formulation for 4π rotation for different load increment schemes.The required moment is applied in 20 steps, 8 steps and 6 steps.In all the cases the magnitude of the incremental rotation is kept less than π to avoid singularity.

Torsion of a flat plate strip

A flat plate strip clamped at one end and subjected to a torsional moment at the free end is another benchmark problem [2]. The plate is twisted for a complete rotation of π. The purpose of this example is to do demonstrate the ability of the formulation to capture large rotations and displacements. The selected properties are $E = 6.825 \times 10^7 N/m^2$ and ν=0.3, $L = 1.0m$, width= $0.25m$ and thickness = $0.1m$. The initial and deformed mesh configuration with 9-noded(1×10) elements are shown in Figure 2. A torsional moment of $3050Nm$ leads to a torsional rotation of about 90^o.It is interesting to note that the converged solution is obtained using only 4 load steps.



(a) Deformed configurations (b) Tip displacements.

Figure 1: Roll-up of a clamped cantilever

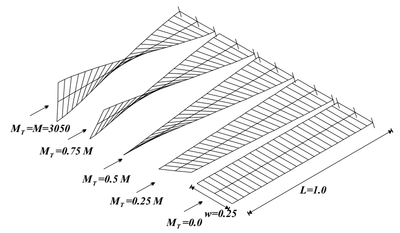
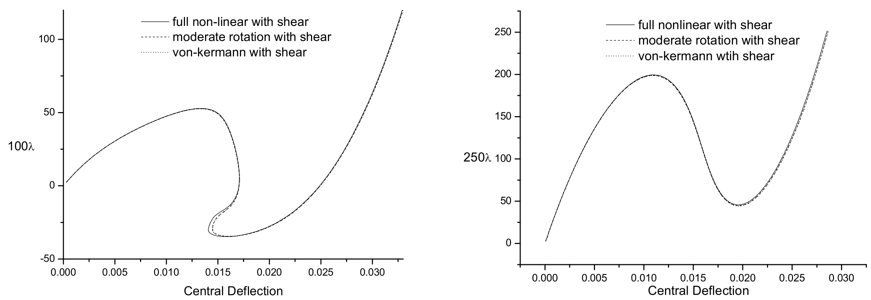


Figure 2: Torsion of a flat plate strip

Snap-through of cylindrical panel

Elastic cylindrical shell panel under central concentrated load is a relevant problem for testing the element performance. This problem was tested by several researchers [3] using a von-Karmann type shell theory.To solve this problem a modified form of Crisfield's arc-length method has been employed. The geometric and material properties are $E = 3105N/mm^2, \nu = 0.3, L = 504mm, R = 2540mm$ and $t = 0.6.35mm, 12.7mm$. The target load has been taken as $1kN$. Figure 3 shows the excellent predictive capability of the present element.



(a) Cylindrical shell panel($t = 6.35$mm) (b) Cylindrical shell panel($t = 12.7$mm).

Figure 3: Snap-through of cylindrical panel

CONCLUSIONS

In this paper we have presented a 5/6 DOF doubly curved laminated shell element for geometrically non-linear problems that accounts for the finite rotation. The finite element matrices have been constructed following the tensorial approach Due to the stress resultant based general shell formulation, the computational time for calculating the element matrices is reduced to a considerable amount. Thus the total computational time reduces compared to that for a degenerated type shell element. The transverse shear and membrane locking problems have been alleviated by implementing the EAS and ANS scheme.The range of applicability of the present element covers arbitrary large displacements and rotations or either thin or moderately thick shells under different load. The excellent predictive capability of the present element shows the applicability of total Lagrangian format in arbitrarily large displacement and rotation problems.

References

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