APPLICATION OF THE RETURN MAPPING ALGORITHM TO PERZYNA VISCOPLASTICITY FOR PLANE STRESS

Zdzislaw Nowak*, Krzysztof Wisniewski*
*Polish Academy of Sciences, IPPT PAN, Swietokrzyska 21, PL 00-049 Warszawa, Poland

Summary In the paper the elastic-viscoplastic constitutive relations of the Perzyna-type is investigated for plane stress problems. A fully implicit integration algorithm is adopted and the relevant expression of the consistent tangent operator for the von Mises yield criterion and flow functions of arbitrary type is derived. It is shown how the elasto-plastic rate equations of standard plasticity can be generalized to overstress-type models of viscoplasticity, where the stress point can be located outside the loading surface. Numerical example is given to reveal the differences and the similarities between the plastic and the viscoplastic overstress models.

PROBLEM FORMULATION

This paper presents a version of the return-mapping algorithm for plane stress problems. Three-dimensional radial return algorithm can be easily modified for the plane strain problem, but not for problems with additional constraints on stresses. Two such constraints are of interest for plate and shells:(a) the zero normal stress condition, $\sigma_{33} = 0$, and (b) the plane stress condition, $\sigma_{\alpha 3} = \sigma_{33} = 0$. With these constraints the classical return-mapping formulation for three-dimensional and the plane strain case is not valid, and an explicit form for plastic multiplier cannot be obtained.

In literature, various viscoplastic material models have been proposed for the analysis of time-dependent deformations in materials. A widely-used viscoplastic formulation is the Perzyna model[1]. The main feature of this model is that the rate-independent yield function used for describing the viscoplastic strain can become larger than zero, which effect is known as 'overstress'. The characteristics of the Perzyna model as well as the numerical discretization have been addressed by various authors (e.g. Simo and Hughes[2]).

PERZYNA MODEL

The main idea of the viscoplastic flow mechanism is to accomplish in one model the description of behaviour of material valid for the entire range of strain rate changes. To achieve this aim the empirical overstress function Φ has been introduced and the strain rate is postulated in the form as follows (cf. Perzyna[1])

$$\dot{\epsilon}_{ij}^{vp} = \gamma \ \langle \ \Phi(\frac{\sigma_{eq}}{\sigma_Y(\epsilon_{eq}^{vp})} - 1) \ \rangle \ \frac{\sigma_{ij}^{'}}{\sigma_{eq}} \tag{1}$$

where σ_{eq} is effective stress, σ_Y is a yield stress and strefunction $\Phi(\cdot)$

$$\Phi(\cdot) \ = \ (\frac{\sigma_{eq}}{\sigma_Y(\epsilon_{eq}^{vp})} - 1)^m \tag{2}$$

In equation (2) m is material constant.

THE ABAQUS FINITE ELEMENT METHOD

In order to implement the Perzyna viscoplasticity model into computer, the return mapping algorithm were implemented in ABAQUS via a user-defined material subroutine UMAT. We consider thin rectangular strip with a circular hole in its axial direction, subjected to increasing extension in a direction perpendicular to the axis of the strip and parallel to the long side of the rectangular section. For symmetry reasons the analysis is performed for one quarter of the section with appropriate boundary conditions. The adopted mesh consists of 415 nodes and 1324 elements. The material used was an aluminium alloy with elastic modulus $E = 70$ GPa, Poisson's ratio $\nu = 0.3$ and a yield stress $\sigma_y = 243$ MPa. Different velocities $\Delta u/\Delta t$ of the imposed displacement are obtained by changing the time increment Δt relative to the single step. The results of the strain and stress distribution in strip are compared with the experimental data published in paper Theocaris and Marketos [3].

CONCLUSIONS

A non-linear solution procedure for rate dependent Perzyna-type models is presented in the context of the backward-Euler method and return-mapping. The plane stress return-mapping formulation for a general overstress viscoplasticity models are described. The main advantages of using the return-mapping algorithm for the viscoplastic models are:

- the stresses are computed in an afficient and explicite way;
- an efficient approach to handle the plane stress case is available;
- the stress updating formulation is significantly simplifies.

It is demonstrated that the present algorithm provides a robust numerical implementation of the viscoplastic model for the plane stress problems. The derived consitent tangent operator gives the appropriate convergence rate in the Newton-Raphson solution procedure. Our numerical experience carried out for a typical benchmark problem and comparison with the experimental data of Theocaris and Marketos [3] for thin perforated strip have been finally shown the accuracy and robustness of the proposed algorithm.

References

[1] Perzyna, P.: Fundamental problems in viscoplasticity, Advances in Applied Mechanics, 9, 343–377, 1966.
[2] Simo J.C., Hughes T.J.R., Computational inelasticity, Springer, New York, 1998.
[3] Theocaris P., Marketos E.: Elastic-plastic analysis of perforated thin strip of a strain-hardening material, J. Mech. Phys. Solids, 12, 377-390, 1964.