

BEAM THEORY FOR ANALYSIS OF GIRDERS WITH CORRUGATED STEEL WEBS

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Summary An extended shear deformable beam bending theory for analysis of girders with corrugated steel webs is derived in this study. The theory is basing on two displacement fields and the assumption on zero longitudinal stiffness of the corrugated steel web. Numerical example on a box-section cantilever analyzed by the theory shows good predictions comparing with the results from the 3D finite element analysis.

INTRODUCTION

Prestressed concrete girder with corrugated steel webs, PCGCSW, is a new concrete-steel composite structural element that has been recently utilized in the superstructures of highway bridges in France and Japan. In the girder, the corrugated steel webs and concrete flanges are separately resisting to shearing force and bending moment, respectively. Consequently, considerable shear deformation in the web and large relative longitudinal displacements of the upper and lower flanges are anticipated. This incurs warping in the cross section of the PCGCSW and invalidates the assumption on plane section remaining plane in the classical beam bending theories.

In this study, an extended shear deformable beam bending theory (abbreviated as *G2-theory*) for analysis of the PCGCSW is derived. The displacement fields utilized in this study are the same as those proposed by Kato et al (2002) [3], however, with different set of governing equations, and the theory is also different from that previously proposed by the authors [4]. Here, the equilibrium equations and boundary conditions of the theory are derived by the principle of minimum potential energy.

AN EXTENDED SHEAR DEFORMABLE BEAM BENDING THEORY (THE G2-THEORY)

Crucial to the development of the theory is the assumption on zero longitudinal axial stiffness in the corrugated webs which implies that the bending moment is entirely resisted by the upper and lower concrete flanges [2]. Thus, the moment-induced normal strain and stress appear only in the flanges. The location of neutral axis of the girder can then be determined only by the first moment equation regarding the upper and lower flanges. From this reason, the longitudinal displacement fields in the G2-theory can be defined basing on the relative displacements of the upper and lower flanges which are, in turn, characterized through a rotational fields (ψ) of the girder. On the other hand, the rotations of the upper and lower flanges are defined equal to the first derivative of the vertical deflection of the girder (dw/dx). The rotational displacement field (ψ), including vertical deflection (w) of the girder are as shown in Figure 1, with dimension of cross section as shown in Figure 2. Here, x -axis designates the longitudinal direction of the beam and z -axis is its transversely downward direction.

By considering that the upper and lower flanges possess only compression-extension modulus of elasticity (E_1) and the web with only reduced shear modulus of elasticity (G_0) [3, 5], the total potential energy functional can be derived, and by applying the principle of minimum potential energy, the following equilibrium equations can be obtained (\bar{q} is transverse load):

$$\bar{q} = e_1 \frac{d^4 w}{dx^4} - \chi^2 g_0 \left(\frac{d^2 w}{dx^2} - \frac{d\psi}{dx} \right) \quad \text{and} \quad 0 = e_0 \frac{d^2 \psi}{dx^2} + \chi^2 g_0 \left(\frac{dw}{dx} - \psi \right), \quad (1,2)$$

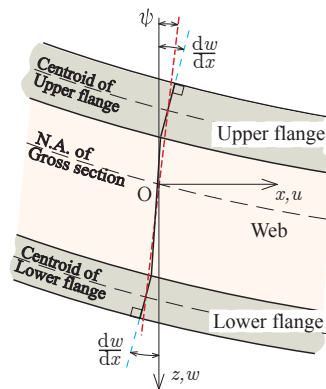


Figure 1: Displacement fields of the G2-theory.

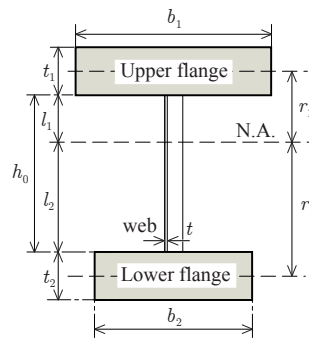


Figure 2: Section of the beam.

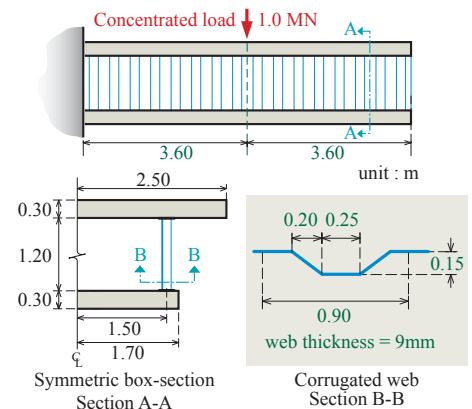


Figure 3: Box-section cantilever.

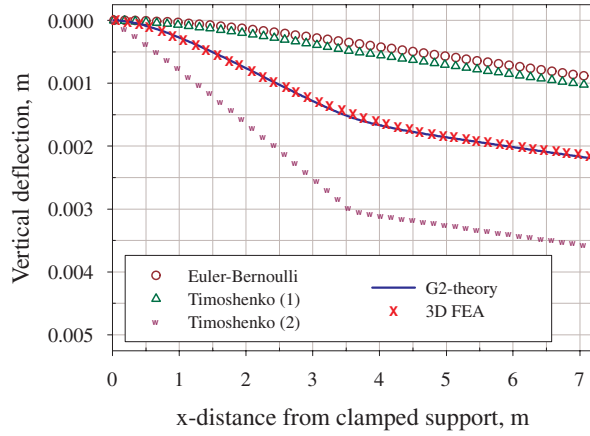


Figure 4: Deflection of box-cantilever.

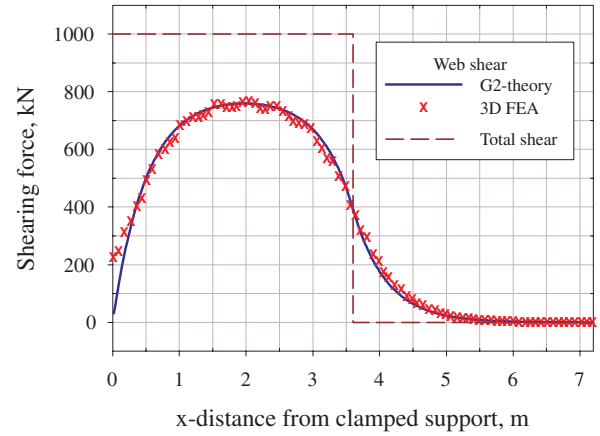


Figure 5: Shearing force in web of box-cantilever.

and the boundary conditions (specific ones of the following conditions):

$$w \quad \text{or} \quad Q_{\text{total}} = Q_{\text{web}} + Q_{\text{flanges}} = \left\{ \chi \mathbf{g}_0 \left(\frac{dw}{dx} - \psi \right) \right\} + \left\{ \rho \chi \mathbf{g}_0 \left(\frac{dw}{dx} - \psi \right) - \mathbf{e}_1 \frac{d^3 w}{dx^3} \right\}, \quad (3)$$

$$\psi \quad \text{or} \quad M_{\psi} = -\mathbf{e}_0 \frac{d\psi}{dx}, \quad (4)$$

$$\frac{dw}{dx} \quad \text{or} \quad M_{dw} = -\mathbf{e}_1 \frac{d^2 w}{dx^2}, \quad (5)$$

here, $M_{\text{total}} = M_{\psi} + M_{dw}$, and

$$\chi = \rho + 1, \quad \chi = \frac{r_1 + r_2}{l_1 + l_2}, \quad \rho = \frac{t_1 + t_2}{2(l_1 + l_2)},$$

$$\mathbf{e}_0 = E_1 (b_1 t_1 r_1^2 + b_2 t_2 r_2^2), \quad \mathbf{e}_1 = E_1 \left(b_1 \frac{t_1^3}{12} + b_2 \frac{t_2^3}{12} \right), \quad \mathbf{g}_0 = G_0 h_0 t.$$

The second term in the last expression of Eq. (3) shows that the shearing force in the upper and lower flanges can not be simply calculated by the $Q = -EI d^3 w / dx^3$ relation, but the web-deformation related shear term must be added too.

NUMERICAL EXAMPLE

The G2-theory is applied to predict vertical deflection and shearing force in corrugated web of a box-section cantilever centrally loaded as shown in Figure 3. The girder is assumed to have $E_1 = 31.0$ GPa, $G_0 = 76.9$ GPa, and also preserve the condition of plane section remaining plane at the free end. The results predicted by the G2-theory are shown in Figure 4 and Figure 5. Also shown in the figures are predictions by the Euler-Bernoulli beam theory, the Timoshenko beam theory considering (1) the whole section and (2) only the web effective in resisting shearing force. They are also compared with the results from the 3D finite element analysis program, ABAQUS (shear lag in flanges are prohibited) [1]. As can be seen in the figures, the G2-theory predicted both the vertical deflection and the web shear of box-cantilever close to the calculated results by the 3D finite element analysis. Shear lag in the web is also predicably by the G2-theory.

CONCLUSIONS

An extended shear deformable beam bending theory specially developed for analysis of the PCGCSW was presented. The theory is proved to provide good prediction for the bending behavior of the PCGCSW comparing with the 3D finite element analysis. The theory can also explain shear lag phenomenon in the corrugated web of the girder.

References

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