

Nonlinear Forced Vibration Analysis of Rectangular Plate Using Super Elements

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SUMMARY: Implementation of finite element methods in the analysis of engineering structures has been widely used in the recent years. In the case of large structures due to the application of a large number of elements and nodes, the method become unattractive. Design of new techniques and elements have been one of the important issues among the finite element engineers. In this regard, dynamic behavior of geometric nonlinear plates under external harmonic force, using **super elements analysis** is considered. A one-quarter model of the plate is proposed to reduce the runtime. Von Karman strain-displacement relations is used. The findings indicate using a few super elements, the same results can be obtained by a large number of regular elements. The computational model is a simple and efficient way to predict the dynamic behavior of the plate. The time required of dynamic analysis is significantly smaller than the regular finite element.

INTRODUCTION

Plate construction is found in a wide range of structures, some of which include ships, bridges, aircrafts, storage tanks and steel gates. In some cases the maximum deflection of the plate is equal or larger than the plate thickness, in these cases, the mid-plane stretches and the in-plane tensile stresses will be developed within the plate. As a result, the plates become stiffened and add considerable load resistance to it, while this phenomena may not be predicted by the small deformation theory of plates. For these cases, accurate theories need to predict the state of plate response. The large-deformation theory of plate can account for the realistic plate deformation, in this theory deflection is no longer small in comparison with the thickness but is nevertheless small compared with the other dimensions [1].

Koko and Olson [2] applied a numerical technique for large deflection elasto-plastic analysis of stiffened plates. They have used a super element which is a macro element having analytical shape functions. The large deflection theory of Von Karman and the Von Mises yield criterion associated with flow rule are employed in their research.

In the present paper, the large deformation theory of Von Karman is applied to analyze the forced vibrations of the plate by using super elements.

MODELING

Plate super element and associated displacement fields are presented by Koko and Olson [2].

Governing equations of motion in the structural dynamic system without damping is:

$$[m]\{\ddot{\delta}\} + [k]\{\delta\} = \{f(t)\} \quad (1)$$

where $[m]$ is mass matrix, $[k]$ is stiffness matrix and $\{\delta\}$, $\{\ddot{\delta}\}$ is displacement and acceleration and $\{f(t)\}$ is force vector. Assuming, the nodal force to be in the form of $\{f(t)\} = \{F\} \sin \omega t$, results in the undamped force response $\{\delta\} = \{\Delta\} \sin \omega t$; then:

$$([k] - \omega^2[m])\{\Delta\} = \{F\} \quad (2)$$

where ω is the external force frequency. Equation (2) is nonlinear equation in terms of nodal displacement and must be solved by nonlinear equation solver.

NUMERICAL EXAMPLES

The method is applied to solve static and dynamic forced vibration of isotropic plates. In the following examples, one-quarter of the plate is modeled by one-super element to analyze the dynamic response of plates and results are compared with conventional element.

Static Analysis of Composite Square Clamped Plate Under Point Force at the Center

A square $[0^\circ/90^\circ/0^\circ]$ symmetric cross-ply laminated plate is considered. Assuming a point force F is applied at the center of the plate. The material properties of laminated composite plate are as follow:

$$E_1=25E_2, G_{12}=G_{13}=0.5E_2, G_{23}=0.2E_2, \nu_{12}=0.25$$

The non-dimensional deflection of plate center can be defined by Reddy [3].

$$\varpi = w \frac{E_2 h^3}{F a^2}$$

where ϖ is non-dimensional deflection of the plate, h and a are the plate thickness and side length of the plate, respectively. In this paper non-dimensional deflection of the plate at the center using super element, ANSYS and Navier's solution (CLPT) are compared. Results indicate good agreement can be obtained using super elements in comparison with conventional finite elements and Navier's solution.

Static Analysis of Isotropic Square Clamped Plate Under Point Force at the Center

In this example, a square isotropic plate is considered. Assuming point force F is applied at the center of the plate. Linear and nonlinear response of super element method and ANSYS are compared in the Figure (1).

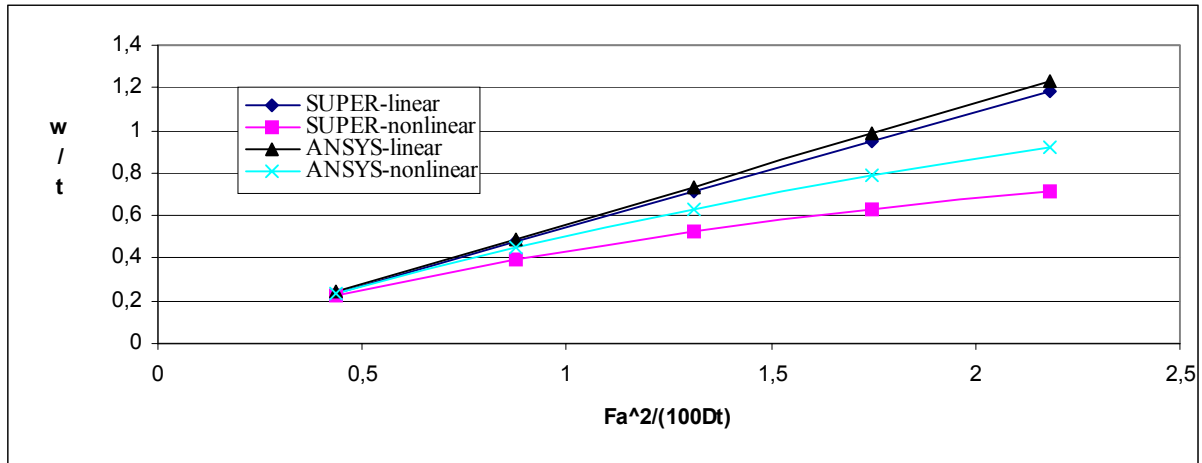


Figure 1. Non-dimensional load-deflection curve of the isotropic plate under the point force F at the center

Forced Vibration Analysis of Isotropic Square Clamped Plate Under Point Force at the Center

In this example, the sinusoidal timing force is applied to the center of the plate. Nonlinear dynamic response of plate in the non-dimensional form is shown in the Figure (2).

Nonlinear response in the non-dimensional form (w/t) versus non-dimensional frequency (ω/ω_1) is shown in this figure (ω is external frequency and ω_1 is linear fundamental frequency).

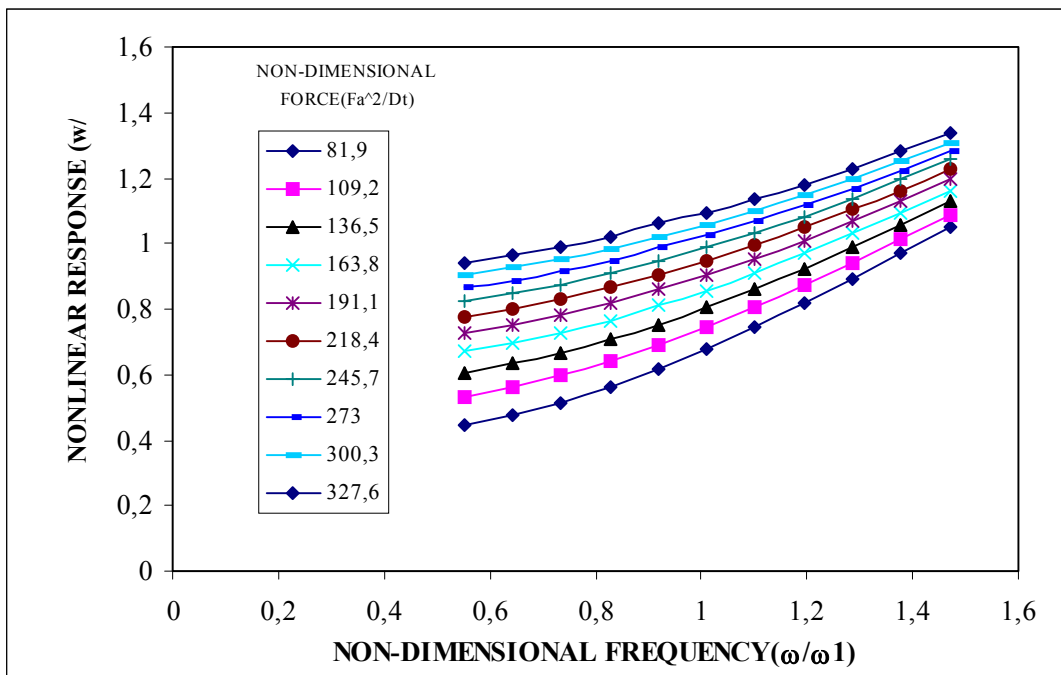


Figure 2. Non-dimensional response of isotropic plate under the point force F at the center

CONCLUSION

Nonlinear finite element analysis of isotropic and symmetric cross-ply laminated composite plates is considered in this paper. Static and dynamic analysis of plate is discussed in the non-dimensional form.

Results indicate application of super element in the nonlinear dynamic analysis of plates give accurate results in less than one-third runtime in comparison with conventional methods.

References

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