NONLINEAR ANALYSES OF RHOMBIC PLATES SUBJECTED TO A CONCENTRATED LOAD

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Summary The theoretical formulations are presented for a rhombic plate subjected to a concentrated load. Several numerical modeling analyses are carried out. Numerical solutions of the present theoretical formulations are shown and compared with the available results using finite element analysis for various plates with different length to width ratios, thickness and supported edges. Some excellent agreements are achieved. It is seen that the accuracy and efficiency of the proposed new theory formulation are confirmed.

INTRODUCTION

Many nonlinear analyses have been presented for the bending problems of rhombic plates. However, in contrast to rectangular, circular and elliptic plates bending, the bending problems of the rhombic plates have not received as much extensive research. This is because there is a very strong singularity at the obtuse vertex along with decreasing of the rhombic angle so that the computational effort increases. It may be relatively difficult to obtain an efficient model to analyze the rhombic plate bending problems. It has showed that the sharp corner singularities in plates and shells might have strong effects upon static or dynamic deflections as the concentrated forces are acted upon plates and shells [1]. For this problem, many efforts have been devoted in previous researches [2-4]. Although current large computational capabilities can improve accuracy of the analysis solution, enormous computational efforts are still required. So far, the numerical techniques such as the finite difference method, finite strip method, dynamic relaxation method, differential quadrature method and finite element method have been adopted for the rhombic skew plate analysis. However, most of the works on rhombic plate bending problems are relevant to the study of small deflection [5-8]. As concerns the large deflection analyses of the rhombic plates bending, there are several early-stage works on theoretical and numerical analyses [9-11]. Unfortunately, all of the above researches, the behaviors of clamped and simply supported isotropic rhombic plates subjected to a uniformly distributed load were investigated merely. The bending problem of the rhombic plate subjected to a concentrated load was always ignored. In fact, there are difficulties for many finite element models to solve such a problem. According to author’s knowledge, the theoretical solution of the rhombic plates bending problem under concentrated has not been proposed. This leads to a difficulty for us to compare which numerical result is more efficient between different methods. The art-of-the-state of research in this field motivates the present study. In this paper, the theoretical formulations of rhombic plates problem are presented. Numerical comparisons were made with the available numerical solutions, and some good agreements were shown.

FORMULATIONS AND NUMERICAL ANALYSES

Large deflection formulations for loosely clamped and simply supported plates are deduced using double cosine series, as follows:

Loosely clamped plate:
$$\frac{P(b/a)^2}{Dh} = \frac{8\pi^4}{c}\left[W_{max} + 0.223545(1-\upsilon^2)W_{max}^3\right]$$

Simply supported plate:
$$\frac{P(b/a)^2}{Dh} = \frac{\pi^6}{4}\left[W_{max} + 0.19476(1-\upsilon^2)W_{max}^3\right]$$

Which *P, D, b/a, h* W_{max}, *v* and c express the concentrated load, the flexural rigidity, the plate length and width ratio, the plate thickness, the maximum deflection, Poisson’s ratio and constant, respectively.

Clamped rhombic plate subjected to a concentrated load

The clamped rhombic plate with a 30^0 angle under a concentrated load *P* at a middle point of the rhombic plate is investigated. In this case, the numerical implementation of finite element analysis software ANSYS [12] is not convergence because of strong singularity. As an available numerical reference, hybrid/mixed finite element analysis (HMFEA) with convergence, reliability and high accuracy is applied [13]. The numerical analyses of HMFEA using 4×4 meshes are performed and applied as reference results. A 30^0 skew rhombic plate ($h/2b = 0.01$) with different length and width ratios of $b/a = 0.667$ and $b/a = 1.0$ are analyzed under a concentrated load (*P*). Load-deflection curves are depicted in Figure 1, which horizontal axis is expressed using a dimensionless term. It is seen that an excellent agreement is obtained. The results of HMFEA show bigger deflection values than the present formulation. This error owes to the numerical analysis of HMFEA.

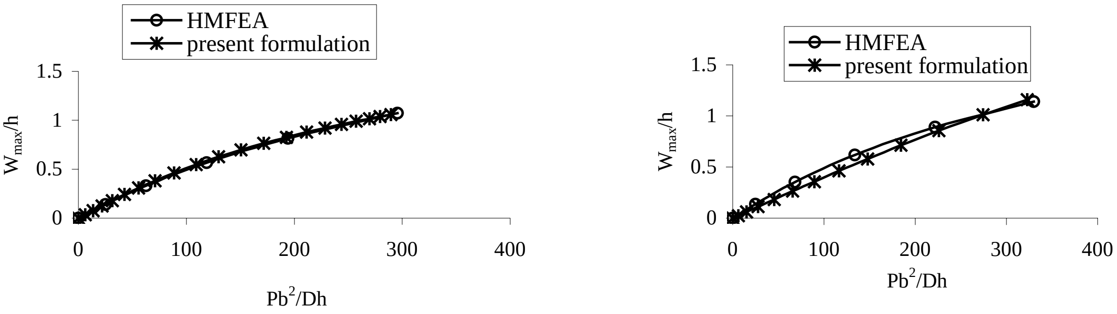

Figure 1 Load-deflection curves of the clamped rhombic plate with 30^0 angle (b/a = 0.667 and b/a = 1)

Simply supported rhombic plate subjected to a concentrated load

The simply supported rhombic plate subjected to a concentrated load is investigated. In this case, 30^0 skew rhombic plates with length and width ratio of $b/a = 1$ and different thickness of $h/2b = 0.01$ and $h/2b=0.1$ are analyzed under concentrated load (P). The numerical results are shown in Fig.2 using dimensionless term in horizontal axis. It is seen that a good agreement is obtained.

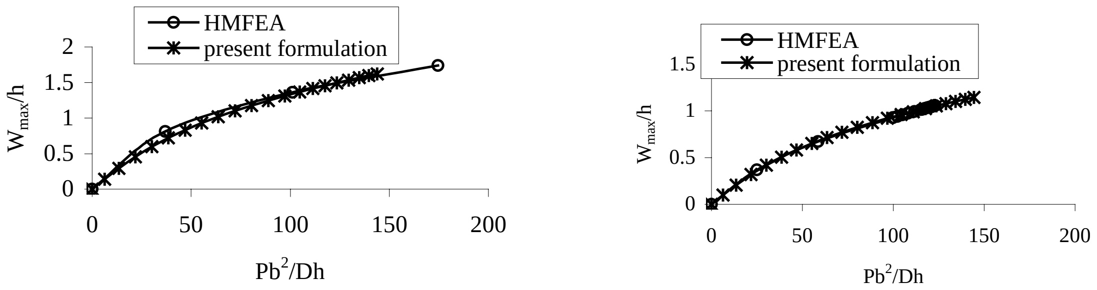

Figure 2 Load-deflection curves of the simply supported rhombic plate with 30^0 angle (b/a = 1, h/2b=0.01 and h/2b=0.1)

CONCLUSIONS

A detailed numerical study on the large deflection behavior has been made for the rhombic plate with different thickness, length to width ratios, various different edges conditions. The numerical results obtained by the proposed formulations have been compared with available reference results. The analyses and accuracy have been verified. The present study provided an efficient theoretical solution for rhombic plates bending problem subjected to a concentrated load.

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