LIMIT AND SHAKEDOWN ANALYSIS WITH DECOHESIVE EFFECTS

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Summary A methodology to predict failure of composite materials is proposed. It is based on local shakedown analysis on the microscopic level of the composite and the use of homogenisation technique to determine the influence of each component (matrix, fiber and interface) on the macroscopic response of such composite.

INTRODUCTION

Local failure under variable loading can be considered as being caused by repeated, dissipative events occurring on the micro-structural level of materials. In this paper, the scale chosen for observation of these effects will be called “meso-scale”, small compared to the scale for measuring the macroscopic dimensions of a mechanical structural element and large compared to the atomistic scale. We suppose that on this mesoscopic level the laws and methods of classical continuum-mechanics are applicable, but that the different components of the material can be recognised, forming a mechanical structure by itself on this level of observation. The interaction between these components determines the local response of the material, in particular the mechanisms leading to local damage and overall failure. Therefore, the study of these mechanisms on the meso-level of specific materials under complex loads can be helpful to better understand the causes of failure and methods developed for this purpose can be used in a constructive manner for the design of materials [1].

A particularly interesting class of materials for this kind of studies are composite materials. Their heterogeneity causes in general large gradients of the mechanical field quantities, initiating local damage and overall failure by the interaction of different local effects, which depend upon the mechanical and geometrical properties of the individual components of the composite. For certain types of composites realistic modelling should take into account not only inelastic behaviour of the matrix material but also debonding of the interfaces between matrix and reinforcements. Special attention has to be paid to the realistic reproduction of the mechanical behaviour of interfaces between matrix and reinforcements, observed in experiments [2]. It is important not only to understand and to model the mechanical effects for this type of materials on the meso-structural level but to link them to the characteristic macroscopic material properties. Here, averaging techniques such as the homogenisation technique [3] are proposed to bridge the gap between local (mesoscopic) and global (macroscopic) properties.

It is shown in this paper, how direct methods, namely limit and shakedown analysis, can help to assess composites which exhibit plastic deformations on the meso-scale and how this theory can be used in a constructive manner for the design of materials. In this contribution, the debonding process is modelled within the framework of interface damage mechanics, where the displacement discontinuities created by the progressive decohesion are related to constitutive equations extended by an anisotropic damage model [4].

DEFINITIONS AND BASIC ASSUMPTIONS

We consider in particular the problem of materials reinforced by straight fibres. The macroscopic behaviour of this heterogeneous material is observed on the scale $\mathbf{x}$ and the mesoscopic behaviour on the scale $\mathbf{y}$. A significant characteristic of fibre-reinforced composites is the limited strength of the interface between fibres and matrix. The micromechanical representative element of total surface Ω presented herein includes the fibre phase, the matrix phase occupying the domains Ω_+ and Ω_-, respectively and the cohesive zone represented by the band-shaped domain Ω_δ of width $\delta << 1$ ($\Omega = \Omega_- \cup \Omega_+ \cup \Omega_\delta$), defined by

$$\Omega_\delta = \left\{ \mathbf{y} = \mathbf{y}_0 + \epsilon \mathbf{n} \;\middle|\; \forall \mathbf{y}_0 \in \Gamma, -\frac{\delta}{2} \le \epsilon \le \frac{\delta}{2} \right\}. \tag{1}$$

The strain field corresponding to the displacement field is derived as

$$\boldsymbol{\varepsilon} = \nabla_s \underline{\mathbf{u}} + ([\![\mathbf{u}]\!] \otimes_s \mathbf{n}\delta_R), \quad \forall \mathbf{y} \in \Omega, \tag{2}$$

where $\underline{\mathbf{u}}$ is the continuous displacement field and $[\![\mathbf{u}]\!]$ is the displacement jump vector, i.e. $[\![\mathbf{u}]\!] = \mathbf{u}^+ - \mathbf{u}^-$. Here, $\mathbf{u}^+$ and $\mathbf{u}^-$ are the displacement vectors at the interior and the exterior borders of the interface zone. $\mathbf{n}$ is the unit vector and δ_R is the Dirac-delta function. For the considered representative volume element (RVE) Ω, we adopt the usual homogenisation assumption for the local displacement field $\mathbf{u}$ at position $\mathbf{y}$

$$\mathbf{u} = \mathbf{E}.\mathbf{y} + \mathbf{u}^{per} \tag{3}$$

where $\mathbf{E}$ is the macroscopic strain tensor and $\mathbf{u}^{per}$ is a displacement field satisfying the periodicity conditions. Then, the Hill relationship [5] holds

$$\mathbf{\Sigma}(\mathbf{x}) = <\boldsymbol{\sigma}(\mathbf{y})> = \frac{1}{|\Omega|}\int_{\Omega} \boldsymbol{\sigma}(\mathbf{y})\, d\Omega \text{ and } \mathbf{E}(\mathbf{x}) = <\boldsymbol{\varepsilon}(\mathbf{y})> = \frac{1}{|\Omega|}\int_{\partial\Omega} (\mathbf{u} \otimes_s \mathbf{n})\, dS. \tag{4}$$

Here, $\boldsymbol{\sigma}$ and $\boldsymbol{\varepsilon}$ are mesoscopic stresses and strains also satisfying the periodicity condition. Within the unit cell, $\boldsymbol{\varepsilon}$ and $\boldsymbol{\sigma}$ fulfil compatibility and equilibrium conditions, respectively.

FORMULATION OF LIMIT AND SHAKEDOWN PROBLEM

Adopting these definitions and hypotheses, the statical shakedown theorems can be stated as follows [6]:
If there exist a real number $\alpha > 1$, a time-independent field of periodic residual stresses $\boldsymbol{\sigma}^{(r)}$, a time-independent vector of traction forces $\boldsymbol{\tau}^{(r)}$ at the interface and an admissible domain P of macroscopic states of stress $\mathbf{\Sigma}$:

$$P = \{\mathbf{\Sigma} \mid \exists\boldsymbol{\sigma}^{(s)}, \exists\boldsymbol{\tau}^{(s)} ;\ \boldsymbol{\sigma}^{(s)}(\mathbf{y}) \in F(\mathbf{y}),\ \boldsymbol{\tau}^{(s)}(\mathbf{y}) \in G(\mathbf{y})\} \tag{5}$$

then the fibre-reinforced composite material shakes down to the given domain of loading. Safe states of stresses $\boldsymbol{\sigma}^{(s)}$ and traction forces $\boldsymbol{\tau}^{(s)}$ are defined, resp., by

$$\boldsymbol{\sigma}^{(s)} = \alpha\,\boldsymbol{\sigma}^{(c)} + \boldsymbol{\sigma}^{(r)},\ \ \boldsymbol{\tau}^{(s)} = \alpha\,\boldsymbol{\tau}^{(c)} + \boldsymbol{\tau}^{(r)} \tag{6}$$

such that

$$F(\mathbf{y}) = \left\{\boldsymbol{\sigma}^{(s)} \mid f(\boldsymbol{\sigma}^{(s)}, \mathbf{y}) < 0,\ \forall\mathbf{y} \in \Omega_{-} \cup \Omega_{+}\right\} \text{ and } G(\mathbf{y}) = \left\{\boldsymbol{\tau}^{(s)} \mid g(\boldsymbol{\tau}^{(s)}, \mathbf{d}, \mathbf{y}) \le 0,\ \forall\mathbf{y} \in \Omega_{\delta}\right\} \tag{7}$$

Here, $\boldsymbol{\sigma}^{(c)}$ and $\boldsymbol{\tau}^{(c)}$ are respectively, the stress field and the traction forces in a purely elastic representative volume element under the same boundary conditions as for the RVE of the original problem, $\boldsymbol{\sigma}^{(r)}$ and $\boldsymbol{\tau}^{(r)}$ are the fields of the residual stresses satisfiying the periodicity conditions and residual traction forces, resp., f is the yield function and g the inelastic gap potential defined by

$$g(\boldsymbol{\tau}^{(s)}, \mathbf{d}, \mathbf{y}) = \left[\left(\frac{\tau_n^{(s)}}{1-d_n}\right)^2 + \frac{1}{\beta^2}\left(\frac{\tau_t^{(s)}}{1-d_t}\right)^2\right]^{1/2} - \sigma_{max} \le 0,\ \ \forall\mathbf{y} \in \Omega_{\delta} \tag{8}$$

Here, σ_{max} is a measure of bond strength of the cohesive zone and β is the interface shear-to-normal strength ratio. The variables d_n and d_t are respectively the normal and the tangential damage variables.

CONCLUSIONS

The presented methodology shows how to apply direct method more realistically to composites by including the process of decohesion between fibres and matrix. The addressed failure mechanism is taken into account by using the concept of a cohesive zone between fibre and matrix. Also, failure due to an unlimited accumulation of the inelastic deformation in the matrix is accounted for. The developed method is applied practically by using finite element analysis and mathematical programming. Illustrative examples will be presented.

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