PARAMETERS IDENTIFICATION OF VISCOPLASTIC MODELS USING EVOLUTIONARY ALGORITHMS

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Summary
A method for parameters identification of different viscoplastic models is presented. The inverse problem is solved using evolutionary algorithms. The metal alloys are investigated in the numerical examples. The results of standard tension tests at different strain rates are used. The identification is applied to the viscoplastic law of Bodner-Partom, Chaboche and Perzyna.

INTRODUCTION
Many technically significant engineering problems should take into account the material hardening and the viscosity phenomena, particularly in the case of extreme loading conditions. The dynamics modeling demands a precise description of material behavior. Different viscoplastic constitutive models are proposed in the literature. They involve numerous parameters. The experimental determination of these material constants can be very complicated for the whole range of possible applications (e.g. at high deformation rates), and often demands sophisticated and expensive conditions. In practice, only a limited number of experimental tests can be provided, carried out for a restricted range of velocities and temperatures. In this work the material parameters of different viscoplastic laws are determined from standard experimental data only. The problem is solved using the evolutionary algorithm. The identification of parameters of metal alloys is presented in the numerical examples. The effectiveness of the approach is studied for the constitutive models of Bodner-Partom, Chaboche and Perzyna.

IDENTIFICATION OF PARAMETERS OF VISCOPLASTIC LAWS
Different models of viscoplastic constitutive laws have been proposed, following experimental investigations on many materials. Among these theories, we can distinguish ones depending on a yield condition or not. The formulations proposed for example by Perzyna or Chaboche, postulate an explicit yield condition as an integral part of the constitutive equations allowing for an easy distinction between linear-elastic and inelastic behavior. Other group of laws (e.g. Miller, Bodner-Partom [1], Walker, Krempl) does not use this concept to separate the elastic domain from the inelastic one, and the same equations can be used directly for all loading and unloading histories.

The viscoplastic constitutive laws are formulated generally as differential equations depending on different material parameters. For instance, the viscoplastic model of Perzyna applies 3 parameters that characterize the viscosity and the initial yield stress. The Chaboche formulation, neglecting recovery effects, uses 7 constants describing the isotropic and kinematic hardening. The Bodner-Partom law [1] needs also 7 constants to precise the strain rate, and the kinematic and isotropic hardening. The process of identification makes use of quantities recorded during experiments. The results of several tests realized in different conditions (e.g. temperatures, loading, strain rates) are often necessary. The determination of the parameters of a constitutive law is essential to characterize the material behavior and to allow precise structural computing.

In this study the optimization approach is applied to the identification problem. In order to determine N parameters $c_1, c_2, \ldots, c_N$ of a given viscoplastic law we dispose of K experimental curves (i.e. stress-strain relations each represented by M points) obtained for different strain rates. The material parameters can be determined by minimizing the difference between the experimental data and the curves simulated numerically. The objective function of the corresponding optimization problem can be expressed as

$$f(c_1, \ldots, c_N) = \sum_{k=1}^{K} \sum_{i=1}^{M} \left[ \sigma_{k,j}^{\text{EXP}} - \sigma_{k,j}^{\text{NUM}}(c_1, \ldots, c_N) \right]^2 \rightarrow \min$$

where $\sigma_{k,j}^{\text{EXP}}$ is the experimental stress value in the i-th point related to the strain $\xi_{k,j}^{\text{EXP}}$ (at the strain rate k), and $\sigma_{k,j}^{\text{NUM}}(c_1, \ldots, c_N)$ is the “numerical” stress value calculated at $\xi_{k,j}^{\text{EXP}}$.

APPLICATION OF EVOLUTIONARY ALGORITHM TO IDENTIFICATION PROBLEM
Evolutionary algorithms (EA) are stochastic search methods inspired by natural evolution and naturally adapted to solve unconstrained optimization problems [2]. They process at a time a number of potential solutions called population and simulate the evolution mechanism using random parameters and biologically inspired operators of crossover, mutation and reproduction. Individuals of a population compete with each other to pass to next generations but only the most suited elements “survive”. The performance of a potential solution is evaluated using the value called fitness. The
process of simulated evolution continues for several iterations and the solution is given by the best-found design, corresponding to the chromosome of maximal fitness value.

The EA developed for this study, uses the floating point, natural representation of design variables. The chromosome of an individual is a string of N material constants $c_i$ $(i=1,\ldots,N)$ and the following fitness definition is applied:

$$\text{fitness}(c_1,\ldots,c_N) = 1/[1 + f(c_1,\ldots,c_N)]$$

(2)

In order to calculate the fitness of each individual, the $\sigma$-$\varepsilon$ curves are simulated numerically (at K strain rates). They apply the values of $c_1,\ldots,c_N$ generated by the EA (within a given range defined by the lower and the upper limit). The solution generated by EA minimizes the errors between experimental and numerical points (a curve fitting problem).

**NUMERICAL EXAMPLES**

The examples below present the parameters identification of a steel, based on the result of one $(K=1)$ and two experimental tension tests performed at different strain rates $(K=2)$. The Bodner-Partom model of material behavior is investigated at room temperature without the cinematic hardening effects. In this case four material parameters $n, m_1, R_0, R_1$ [1] are searched. First, the Young modulus $E=220000$ MPa, and the deformation rates 0.0058 and 0.059 of two experiments have been determined numerically by the linear regression method from data files recorded during experiments. These values have been used by the procedure simulating numerically $\sigma$-$\varepsilon$ curves during 300 sec. The material parameters have been searched within the following limits, based on the literature values: $0.1 \leq n \leq 20$, $0.1 \leq m_1 \leq 1.5$ (MPa$^{-1}$), $200 \leq R_0 \leq 2000$ (MPa), $100 \leq R_1 \leq 2500$ (MPa). The 25 generations of 40 individuals population have been studied by the Evolutionary Algorithm and the elitist reproduction technique has been applied.

![Fig. 1. One experimental $\sigma$-$\varepsilon$ curve and simulated Bodner-Partom model of steel](image1.png)

![Fig. 2. Two experimental $\sigma$-$\varepsilon$ curves and simulated Bodner-Partom model of steel](image2.png)

On Fig.1 and Fig.2 the experimental points are compared with the material curves plotted for the parameters obtained using the EA identification procedure. A good correlation between the experimental and numerical graphs can be noticed. However, the direct extension of the solution obtained using one experimental test (Fig.1) to simulate higher speed phenomena may reduce considerably the precision. Several experimental tests, carried out at different strain rates (as in the case of the study presented on Fig.2) are necessary to take into account the viscosity effects and the strain rate sensitivity of the material. This aspect of the problem is studied also in other examples realized in this work for the viscoplastic models of Bodner-Partom, Perzyna, and Chaboche. The quality of the results and the robustness of the approach are examined for several independent runs of the optimization procedure.

**CONCLUSIONS AND FINAL REMARKS**

The Evolutionary Algorithm based identification method enables to obtain very satisfactory results even for relatively large number and range of parameters variation. The procedure takes approximately a few minutes of CPU on a PC computer (depending on the number and the time duration interval of the simulated curves). The efficacy of the identification procedure can be enhanced by an appropriate tuning of genetic operators, the limitation of parameters variation range. A hybrid approach, incorporating the specific, problem oriented knowledge (like a good estimation of some material constants, based on the experience) is worth to be investigated in further studies.

**References**
